BASICS
Of
RADIO TELESCOPES

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Introduction

All astronomical measurements are based on the radiation from the source and the power received on Earth. For convenience, we establish a relationship between the power radiated from a location on the sky and corresponding power received on the Earth. Thus a basic unit of radiation is to be developed which should describe the dependence of all the variables involved in radiation. This unit is called ***brightness*** is astronomy. If the brightness is known, then it may be used to calculate the received spectral power etc. on the Earth.

In this lecture, the concept of ***brightness flux density***, ***effect of the antenna beam pattern on observations***, ***blackbody radiation***, ***absorption*** and ***emission***, ***temperature***, ***noise*** and ***sensitivity*** (minimum detectable temperature) and some theorems related to antenna patterns have been described in details.

Comparing Isotropic src. with Astronomical src.

How to specify the term ***Brightness***? Some ideas come from below:

Isotropic Radiator	Radio Astronomy Source
Zero angular extent, or point source. •*Angular size of the source is not required.*	Non-zero angular extent. •*Angular size of the source is required.*
Power is constant over all frequencies. •*Only power input is to be specified.*	Power varies with frequency. •*Power spectrum of the source is required.*
Radiates equally in all directions. •*Gain is uniform in all directions.* •*Uniform radiation intensity in all directions*	Radiates uniformly on all places of Earth. •*Uniform gain towards Earth.* •*Uniform radiation intensity in all directions towards Earth.*

Comparing Isotropic src. with Astronomical src.

Comparison of isotropic radiator with a radio astronomical source gives us the following intutive clues for specifying brightness:

(*i*) Brightness should contain the radio source dimensions (angular extent specified by a solid angle as seen by the observer).

(*ii*) It should contain is spectral information. In other words, it should give us some idea about how its power changes as a function of frequency.

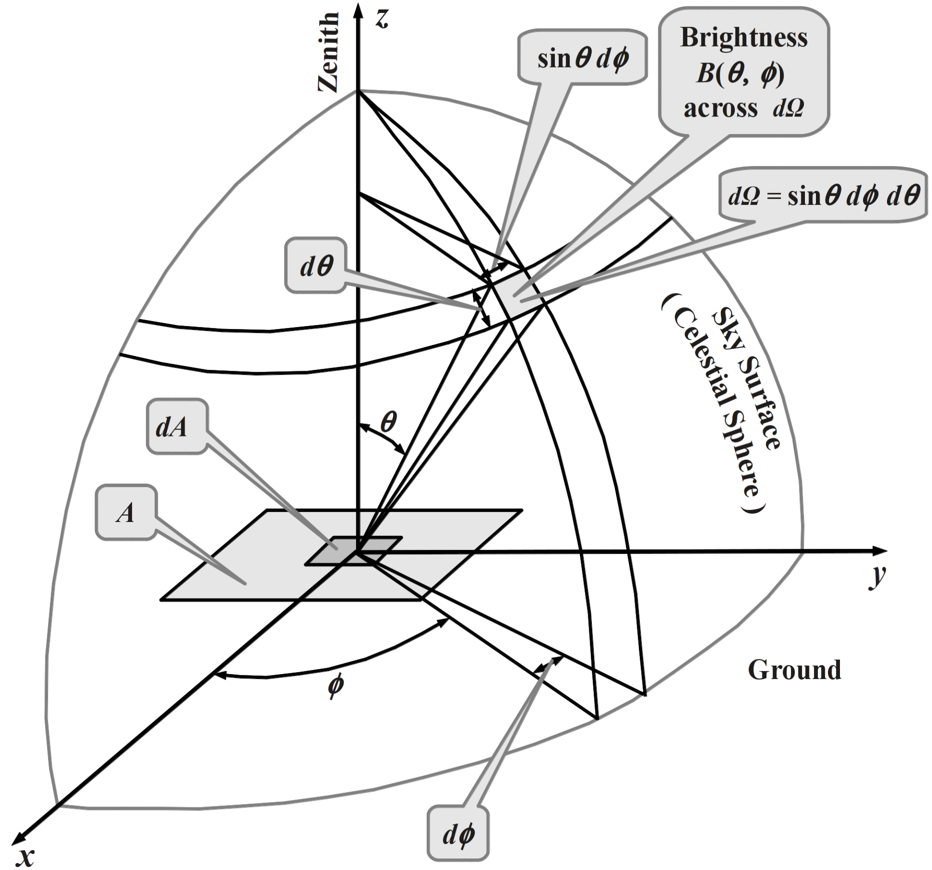
(*iii*) It should also specify how much power it avails at the receiving end (observer) over a specified area.

From above, we may specify the unit of brightness as **power/unit-area/unit-frequency/unit-solid-angle,** e.g., **watts/m^2/Hz/steradian.**

In general, brightness is used to specify the radiations arriving from a portion of the celestial sphere. These radiations can be from cold sky, discrete radio sources or extended radio sources on that portion of sky.

Brightness

Brightness: *Power radiated per unit area per unit frequency per unit solid angle.* It is measured in watts/m^2/Hz/steradian.



An electromagnetic radiation of bandwidth $d\nu$ Hz from a region of the sky (θ,φ) falls on a flat area dA on Earth. An infinitesimal power dW received from the region of the sky subtending a solid angle $d\Omega = \sin\theta \; d\theta \; d\varphi$ and having a brightness variation $B(\theta,\varphi)$ can be expressed as:

$$dW = B(\theta,\phi)\cos\theta \, d\Omega \, dA \, d\nu$$

Thus infinitesimal power received by the surface A over the bandwidth $d\nu$ is given as:

$$dW = A\,B(\theta,\phi)\cos\theta \, d\Omega \, d\nu$$

Brightness Spectrum and Total Brightness

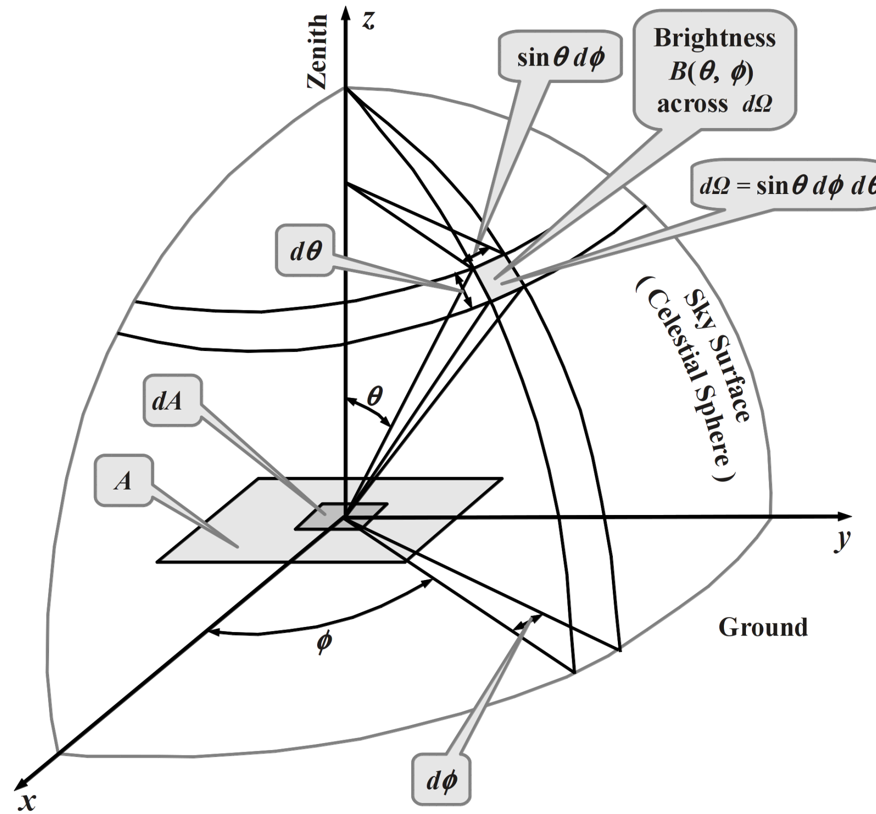
Infinitesimal power received dW by A over bandwidth $d\nu$ is:

$$dW = A\,B(\theta,\phi)\cos\theta\, d\Omega\, d\nu$$

Received power W by A over the bandwidth $\nu+\Delta\nu$ is

$$W = A\int_{\nu}^{\nu+\Delta\nu}\iint_{\Omega} B(\theta,\phi)\cos\theta\, d\Omega\, d\nu$$

The variation of brightness B as a function of frequency is known as ***brightness spectrum***.



Another variety of brightness is the ***total brightness*** *B' obtained by* integrating B over a bandwidth $\Delta\nu$ Hz.

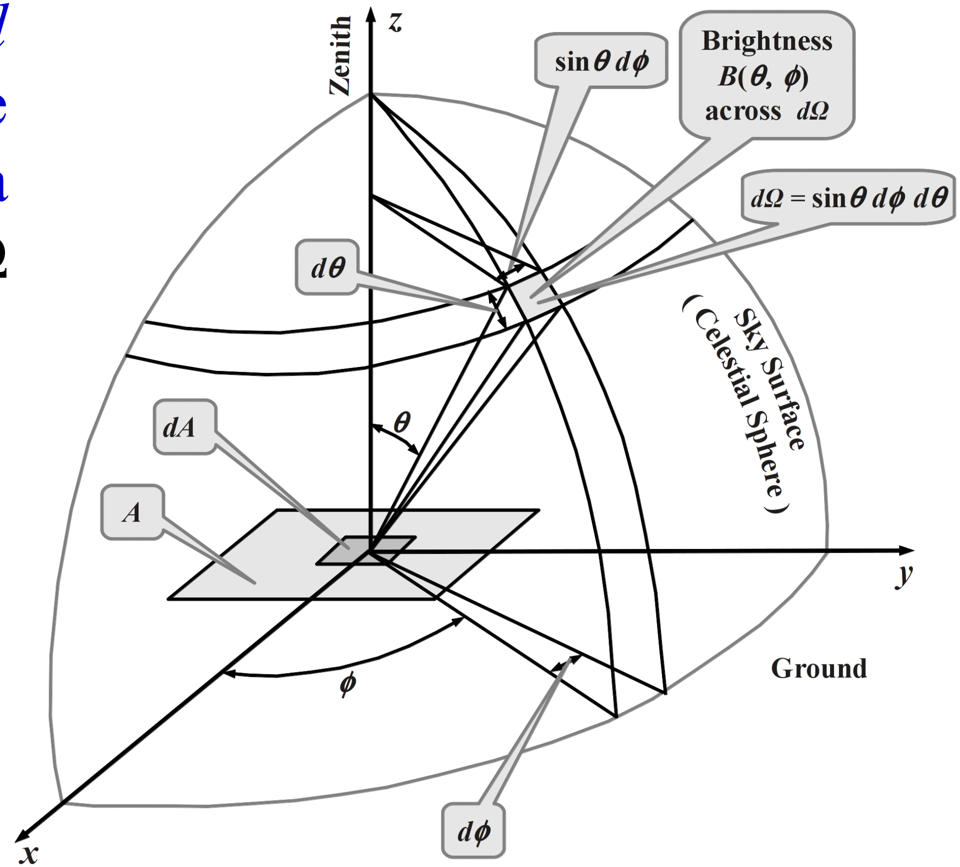
$$B' = \int_{\nu}^{\nu+\Delta\nu} B(\theta,\phi)\, d\nu$$

Yet another variety of brightness is the ***total radio brightness*** which is obtained by integrating brightness over the entire radio spectrum.

Spectral Power

Using the concept of *total brightness* B', we can express the received power W over a bandwidth $\Delta\nu$ and solid angle Ω as:

$$W = A \iint_{\Omega} B'(\theta, \phi) \cos\theta \, d\Omega$$



Note that we are no more integrating it over any bandwidth.

The power per unit bandwidth is called as ***spectral power*** w and is measured in watts/Hz.

Infinitesimal spectral power across dA is $dw = B(\theta,\phi)\cos\theta \, d\Omega \, dA$

Infinitesimal spectral power across A is $dw = A\, B(\theta,\phi) \cos\theta \, d\Omega$

Spectral power from solid angle Ω received across A is

$$w = A \iint_{\Omega} B(\theta,\phi) \cos\theta \, d\Omega$$

Using a Radio Telescope Antenna

The detector collecting the radiation is an antenna, also called ***radio telescope***. Thus, equations have to be modified with the radiation pattern. Let an antenna having an effective aperture A_e and a normalized response pattern $P_n(\theta,\varphi)$ receives the radiations. Then the equation for the spectral power obtained from solid angle Ω received across A given as $w = A \iint_{\Omega} B(\theta,\phi)\cos\theta\, d\Omega$

can be modified to $w = \frac{1}{2}\left(A_e \iint_{\Omega} B(\theta,\phi)\, P_n(\theta,\phi)\, d\Omega\right)$

<u>**Note**</u>**:** Limits of Ω should be selected such that the integration does not miss the nonzero points of $P_n(\theta,\varphi)$. A factor of half has been introduced assuming that the antenna responds to only one polarization.

Thus the total power W received by a radio telescope antenna can be shown as:

$$W = \frac{1}{2}\left(A_e \int_{\nu}^{\nu+\Delta\nu} \iint_{\Omega} B(\theta,\phi)\, P_n(\theta,\phi)\, d\Omega\, d\nu\right)$$

Seeing Discrete Radio Sources

A tiny radio source in some region of the sky called as ***discrete source*** can be categorized into three:

(*i*) Point source – no angular dimension or negligible with respect to antenna beam.

(*ii*) Localized source – small but finite dimension, much less than antenna beam solid angle.

(*iii*) Extended source – larger dimension, comparable to the antenna beam solid angle.

For a discrete radio source, total source flux density is

$$S = \iint_{\Omega_S} B(\theta, \phi)\, d\Omega$$

When seen with antenna

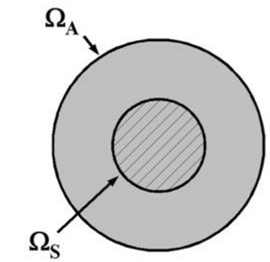
$$S = \iint_{\Omega_S} B(\theta, \phi)\, P_n(\theta, \phi)\, d\Omega$$

If source extent equals major lobe

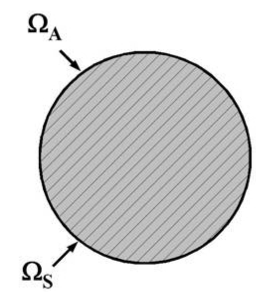
$$S = \iint_{\Omega_S} B(\theta, \phi)\, d\Omega$$

If source extent is greater than major lobe:

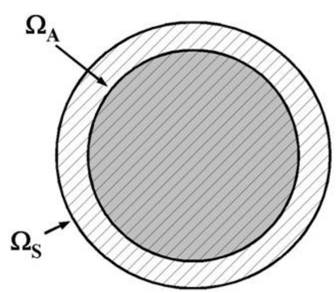
$$S = B(\theta, \phi) \iint P_n(\theta, \phi)\, d\Omega \simeq B(\theta, \phi)\, \Omega_M$$



Source extent < major lobe



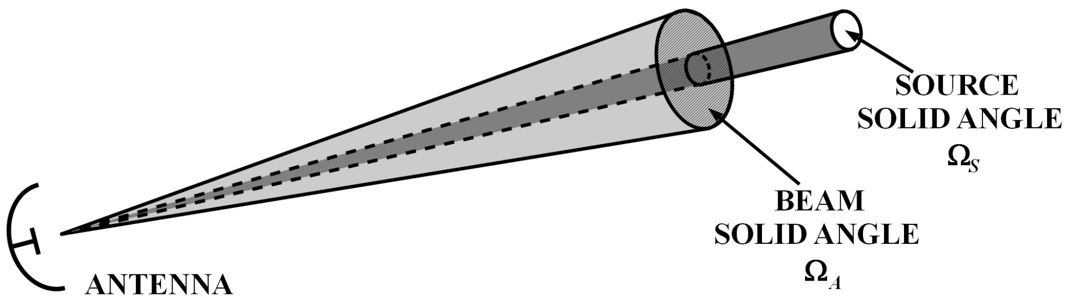
Source extent = major lobe

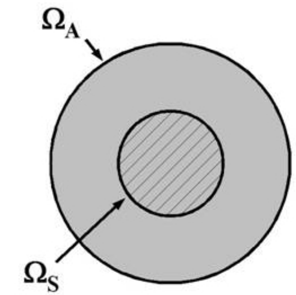


Source extent > major lobe

Seeing Discrete Radio Sources

An antenna beam pointed to center of a radio source.





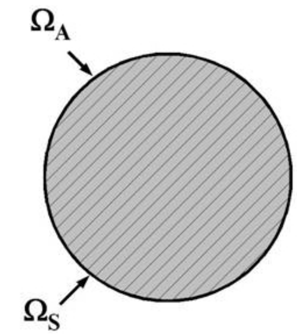
Source extent < major lobe

Thus, there can be three cases of pointing:

(*i*) Solid angle of radio source < antenna beam.

(*ii*) Solid angle of radio source = antenna beam.

(*iii*) Solid angle of radio source > antenna beam.

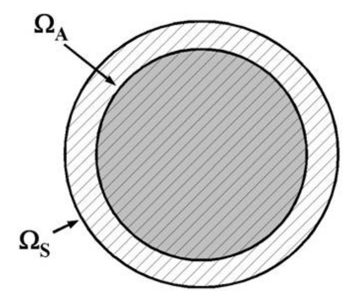


Source extent = major lobe

Antenna pattern is not uniform. Thus observed flux density is less than its true value.

The ***observed*** or ***apparent brightness*** B_e (using an antenna) is:

$$B_e = \frac{\iint B(\theta,\phi)\, P_n(\theta,\phi)\, d\Omega}{\iint P_n(\theta,\phi)\, d\Omega} = \frac{S_0}{\Omega_A}$$



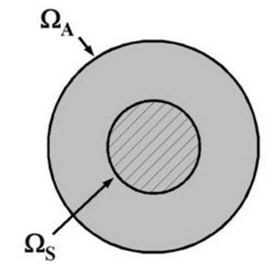
Source extent > major lobe

where, S_0 is received ***flux density*** in watt/m^2/Hz and Ω_A is beam solid angle of antenna.

Seeing Discrete Radio Sources

Case (*i*): Solid angle of radio source is smaller than the antenna beam.

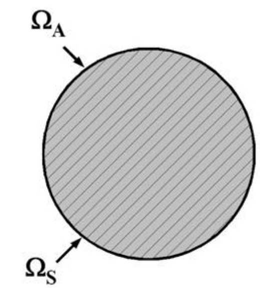
Average brightness $B_{avg} = \dfrac{S}{\Omega_S} = \dfrac{1}{\Omega_S} \iint B(\theta, \phi)\, d\Omega$



Source extent < major lobe

If the source has a uniform brightness across it ($B = B_{avg}$), then the apparent brightness B_e is given as:

$$B_e = \frac{S}{\Omega_A} = \frac{\Omega_S}{\Omega_A} B$$



Source extent = major lobe

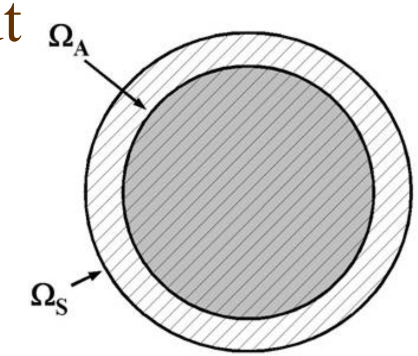
Case (*ii*): When the extents of the source coincide with the first nulls of the antenna beam, the apparent brightness is

$$B_e = \frac{S}{\Omega_A} = \frac{\Omega_M}{\Omega_A} B = \varepsilon_M B \quad \text{where, } \Omega_M = \iint_{\Omega_M} P_n(\theta, \phi)\, d\Omega$$

where, $\mathbf{\Omega}_M$ is the solid angle subtended by the main lobe of the antenna beam and ε_M is beam efficiency (*see* Basics of Antennas).

Seeing Discrete Radio Sources

Case (*iii*): Solid angle subtended by the source is larger than the major lobe of the antenna beam, but occupies a finite region on the sky.



Source extent > major lobe

Apparent brightness

$$B_e = \frac{S}{\Omega_A} = \frac{\Omega_M^{'}}{\Omega_A} B \quad \text{where, } \Omega_M^{'} = \iint P_n(\theta, \phi)\, d\Omega$$

where, Ω'_M is the beam solid angle of the major and side lobes which fall within the solid angular extent of the source.

The relation of the antenna beam solid angles are expressed below. These are termed as ***beam efficiencies*** (*see* Basics of Antennas).

$$\frac{\Omega_M}{\Omega_A} < \frac{\Omega_M^{'}}{\Omega_A} < 1$$

Seeing Discrete Radio Sources

Generally, many of the engineering applications require the flux density calculations in watts/m^2 which is the ***total flux density*** denoted by *S'*. This involves the integration of the flux density *S* within the associated bandwidth of observation.

Total flux density $$S' = \int_{\nu}^{\nu+\Delta\nu} S \, d\nu$$

where, $\Delta\nu$ is the bandwidth of the instrument.

Total power *W* observed over a bandwidth $\Delta\nu$ can be expressed as:

$$W = \frac{1}{2} A_e \, S_0' = \frac{1}{2} A_e \int_{\nu}^{\nu+\Delta\nu} S_0 \, d\nu = \frac{1}{2} A_e \int_{\nu}^{\nu+\Delta\nu} \iint_{\Omega_S} B(\theta, \phi) \, P_n(\theta, \phi) \, d\Omega \, d\nu$$

where,

S_0' = Total observed flux density (using antenna) in watts/m^2.

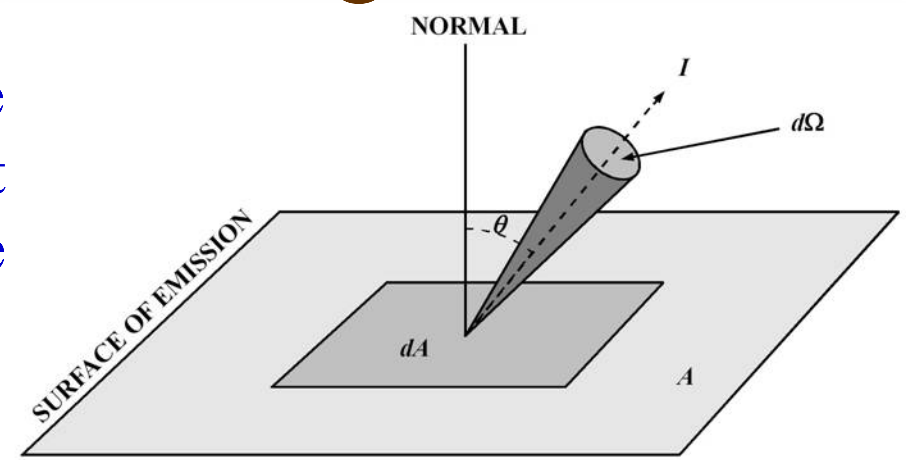
S_0 = Total observed flux density (using antenna) in watts/m^2/Hz.

The total power *W* may be approximated as $W = \frac{1}{2} A_e \, S_0 \, \Delta\nu$

Radiance/Intensity and Brightness

The spectral power escaping the surface element *dA* and coming out through the solid angle $d\Omega$ can be expressed as:

$$dw = I \cos\theta \, d\Omega \, dA$$



where, dw is the spectral power per unit bandwidth in watts/m^2/Hz, I is the intensity or radiation from the surface in watts/m^2/Hz/rad^2, and θ is the angle between the normal to the surface.

I is commonly known as ***radiance*** and has a dimension of power per unit area per unit solid angle per unit bandwidth.

If the radiation from the surface is uniform over the entire area A, then

$$dw = A\, I \cos\theta \, d\Omega$$

Note: Intensity I and brightness B have the same units, except that the term intensity is used in radiation and brightness is used for reception. Hence, $I = -B$.

Governing Laws of Black Body

Any object possessing a temperature greater than 0 K radiates. The wavelength and intensity of radiation are temperature dependent.

A blackbody is a conceptual object which completely absorbs the electromagnetic energy of any wavelength incident on it and does not reflect or lets them pass through it.

The reverse is also true, i.e. *if temperature of the blackbody is non-zero, it can radiate with a similar spectrum.*

The various laws associated with the blackbody are:

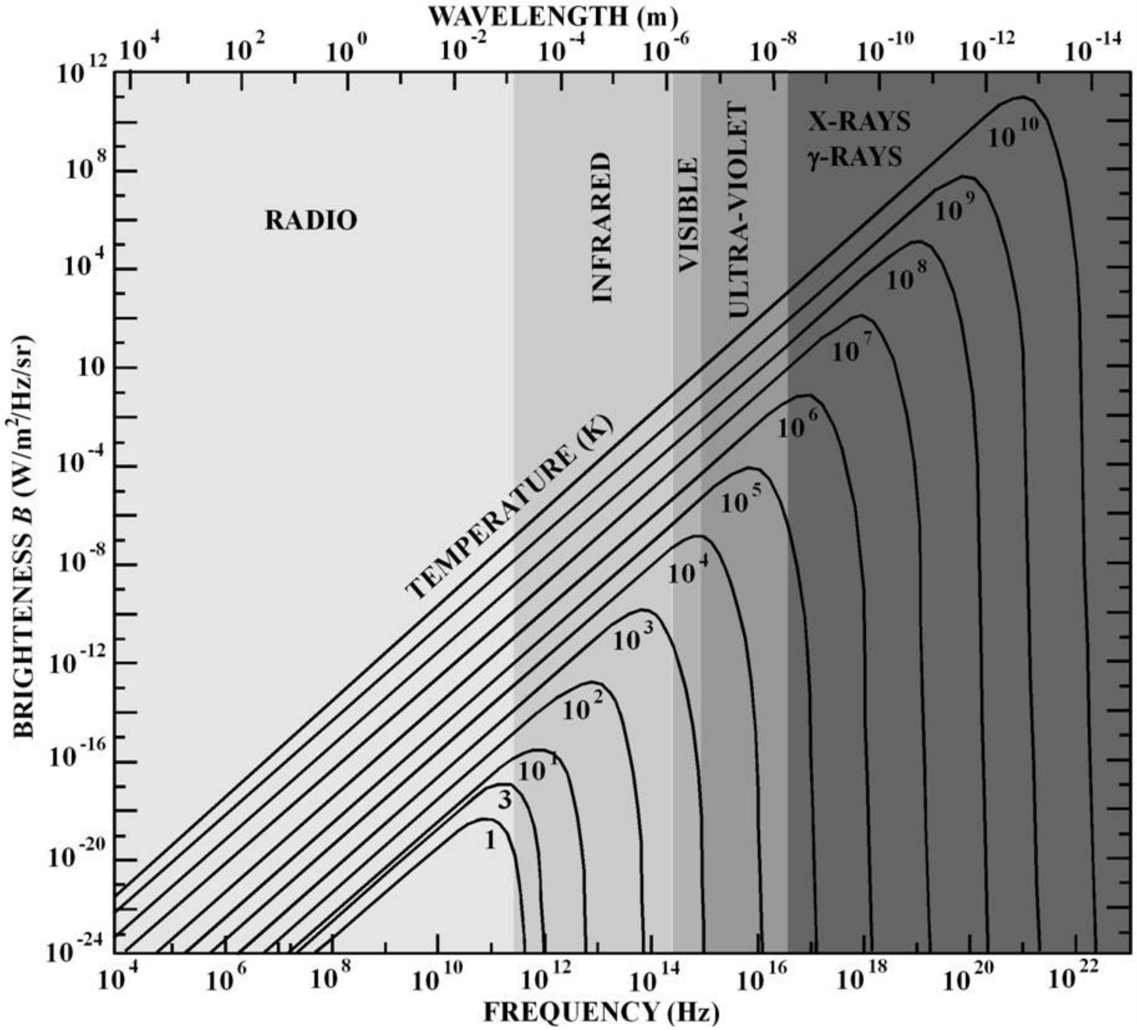
(*i*) Planck's law of Spectral Radiance.
(*ii*) Kirchhoff law of thermal radiation.
(*iii*) Stefan-Boltzmann law.
(*iv*) Wien's Displacement law.
(*v*) Laws approximating Planck's law of Spectral Radiance:
- (*a*) Wien's Radiation law.
- (*b*) Rayleigh-Jeans radiation law.

Black Body: Planck's law

Planck's law: It describes the spectral radiance of electromagnetic radiation at all wavelengths from a blackbody at temperature T as a function of frequency ν or wavelength λ and is expressed as:

$$B(\nu) = \frac{2\,h\,\nu^3}{c^2}\left(\frac{1}{e^{\frac{h\nu}{kT}} - 1}\right)$$

$$B(\lambda) = \frac{2\,h\,c^2}{\lambda^5}\left(\frac{1}{e^{\frac{h\,\nu}{\lambda\,k\,T}} - 1}\right)$$



Boltzmann constant (k) = $1.38\text{x}10^{-23}$ J/K

Planck's constant (h) = $6.6260693\text{x}10^{-34}$ J/Hz

Black Body: Kirchoff, Steffan-Boltzmann

Kirchoff's law: *If a thermal equilibrium is achieved between the blackbody with its surrounding, then this law implies that the emission of the blackbody is equal to its absorption.*

Stefan Boltzmann law: *The total energy radiated per unit surface area of a blackbody per unit time per unit solid angle* (i.e., *total brightness B'*) is directly proportional to the fourth power of the blackbody's absolute temperature *T* and is given as:

$$B' = \sigma_{B'} T^4 \quad \text{watts m}^{-2}\text{rad}^{-2}$$

where, $\sigma_{B'}$ is Stefan-Boltzmann constant for brightness for all wavelengths and is equal to 1.8047×10^{-8} watts/m^2/rad^2/K^4.

For a star of radius *R*, its luminosity *L* is expressed as:

$$\left.\begin{aligned} L &= 4\pi R^2 \sigma T^4 \quad \text{watts} \\ \text{where, } \sigma &= \pi \sigma_{B'} \end{aligned}\right\}$$

where, σ = 5.67x10-8 watts/m^2/K^4.

Black Body: Wien's Displacement Law

Wien's displacement law: *It states that wavelength* λ_{peak} *at which the emission from a blackbody peaks is inversely proportional to the temperature* T *of the object.*

It may be expressed as:

$$\lambda_{peak} = \frac{2.88 \times 10^{-3}}{T} \ \ \mathrm{m}$$

where, λ_{peak} is in meters and T is in K.

Black Body: Approximating Planck's Law

These laws only approximate the radiation from a blackbody at a given temperature over a certain range of frequencies. There two laws are described below.

(*i*) **Wien's radiation law:** It is an approximation of the Planck's law of spectral radiance at higher frequencies and is given as:

$$B = \frac{2h\nu}{\lambda^2} e^{-\frac{h\nu}{kT}}$$

where, B is the brightness of the object.

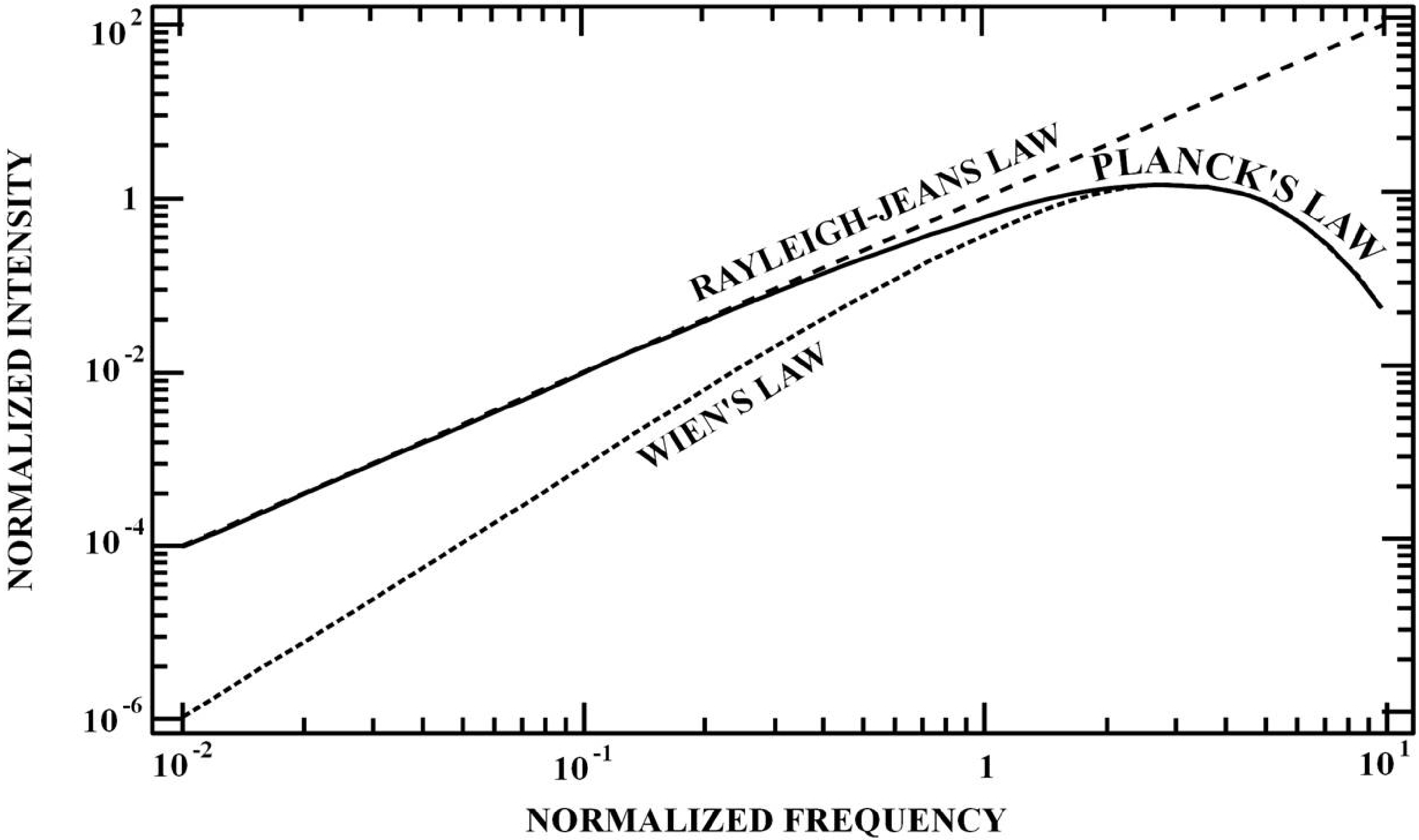
(*ii*) **Rayleigh-Jeans radiation law:** It is an approximation of the Planck's law of spectral radiance at lower frequencies and is given as:

$$B = \frac{2kT}{\lambda^2}$$

where, B is the brightness of the object.

The Wien's and Rayleigh-Jeans laws respectively approximate the higher and lower frequency part of the spectrum obtained using Planck's law of spectral radiance.

Comparison between Approximate Laws



Raleigh-Jeans law and Wien’s laws are compared with Plack’s law.

Propagation: Absorption Emisssion

The electromagnetic waves emitted from a radio source travel an extremely large distance before reaching the Earth. The medium is not exactly an empty space. It contains various gas and dust clouds, for example the interstellar medium. The elements in these gases affect the waves passing through it by producing phenomena like (*i*) absorption, (*ii*) emission or (*iii*) both.

We study the following cases of wave propagation:

(*i*) Effect of Absorption of Electromagnetic Waves.

(*ii*) Effect of Emission of Electromagnetic Waves.

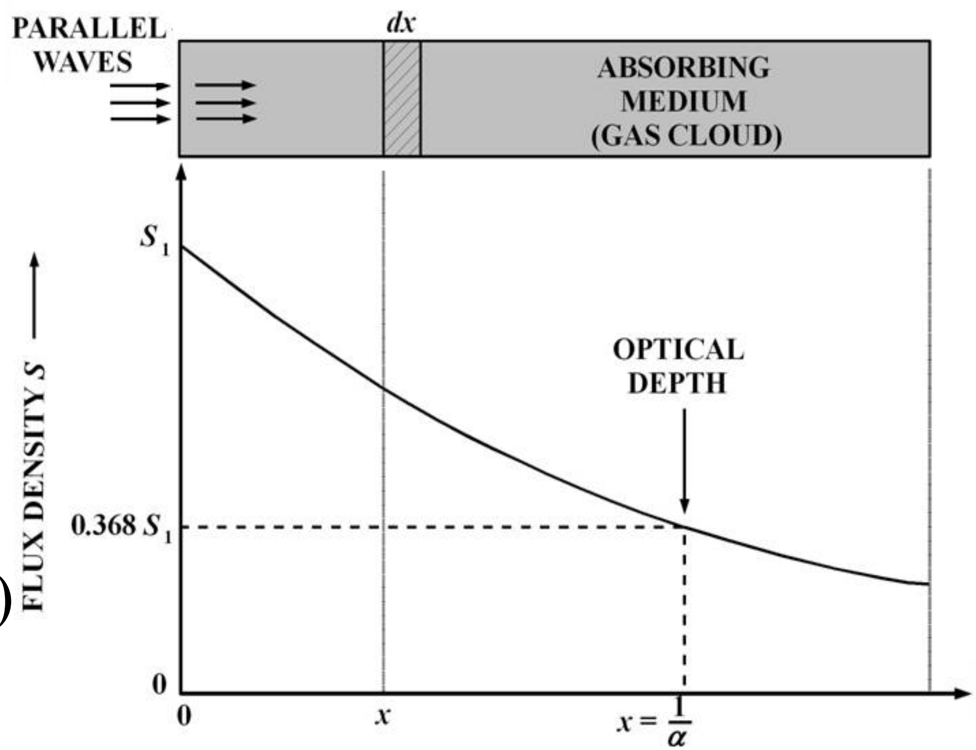
(*iii*) Effect of Absorption and Emission of Electromagnetic waves:

(*a*) Internal Emission and Absorption by a Cloud.

(*b*) External Radio Source observed through an Emitting and Absorbing Cloud.

Effect of Absorption of EM Waves

Consider a small region of length x along the propagation direction in an absorbing medium.



S_1 = Flux density at region's entry

Flux density at region's exit is:

$$S = S_1\, e^{-\alpha x} = S_1\, e^{-\tau}$$

where,

α = attenuation constant (Nep/m)

τ = optical depth (m) given as:

$$\tau = \ln\left(\frac{S_1}{S}\right) = 2.3\ \log\left(\frac{S_1}{S}\right)$$

[τ = (attn. const.) × (layer depth)]

Brightness at exit $B = B_S\, e^{-\alpha x} = B_S\, e^{-\tau}$ where, B_S = brightness at entry.

For gaseous mediums, $\alpha = K\,\rho$ where, K is ***absorption co-efficient*** in m^2/kg and density ρ is in kg/m^3. Optical depth $\tau = \int_0^{x_1} K\,\rho(x)\,dx$

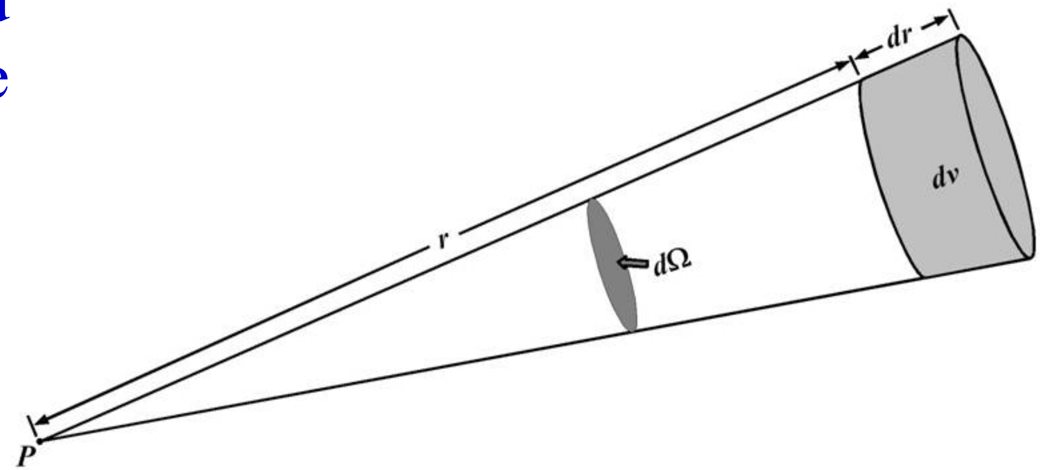
Depth of penetration:

Value of x at which $S / S_1 = 1/e = 0.368$

Effect of Emission of EM Waves

Let a volume of gas dv subtend a solid angle $d\Omega$ with the observer at a distance r.

j = Emission co-efficient (rate of energy emission per unit volume per unit mass per unit bandwidth in watts/kg/Hz).



Infinitesimal flux density observed at P is $dS = \dfrac{dw}{4\pi r^2} = \dfrac{j\,\rho\,dv}{4\pi r^2}$

Infinitesimal brightness observed at P is:

$$dB = \frac{dS}{d\Omega} = \frac{j\,\rho\,dv}{4\pi\,r^2 d\Omega} = \frac{j\,\rho\,dr}{4\pi}$$ since, $dv = r^2\,dr\,d\Omega$

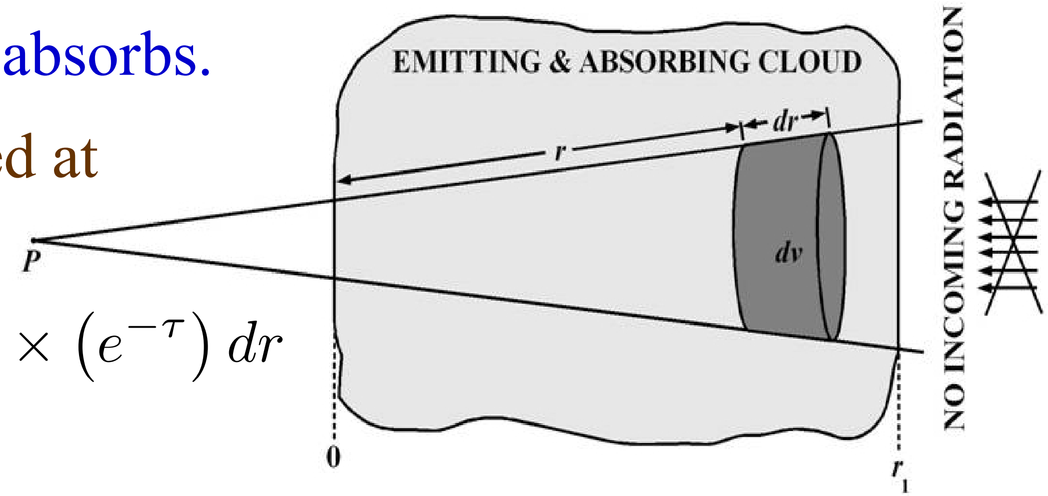
Brightness B for any finite depth of emitting matter enclosed between radii r_1 and r_2 is

$$B = \frac{1}{4\pi}\int_{r_1}^{r_2} j\,\rho\,dr$$

Effect of both Emission and Absorption

(*a*) Internal Emission and Absorption by a Cloud

The volume *dv* emits as well as absorbs.



Infinitesimal brightness observed at point *P* is:

$$dB = dEmi - dAttn = \left(\frac{1}{4\pi} j\rho\right) \times \left(e^{-\tau}\right) dr$$

where, $\tau = \int_0^r K\rho dr$

and *K* is *absorption co-efficient* in m²/kg.

Brightness due to the cloud from 0 to r_1 is $B = \int_0^{r_1} dB = \frac{j}{4\pi K}\left(1 - e^{-\tau_c}\right)$

where,

B = apparent brightness, $\tau_c = \int_0^{r_1} K \rho \, dr$ (cloud's optical thickness)

Intrinsic brightness is $B_i = j / 4\pi K$ Also, $B = B_i\left(1 - e^{-\tau_c}\right)$

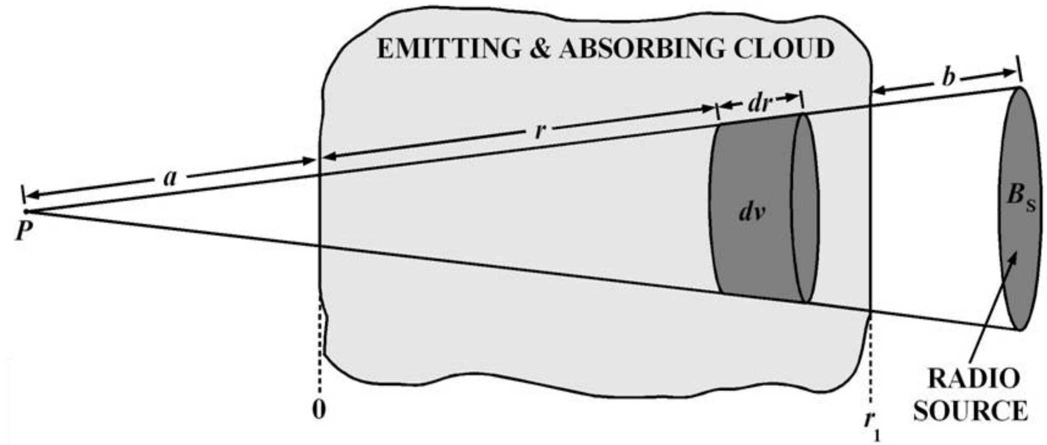
Since *B* is proportional to *T* (Raleigh-Jeans law), we have:

$T_b = T_c\left(1 - e^{-\tau_c}\right)$ where, T_b = observed temp., T_c = cloud's temp.

Effect of both Emission and Absorption

(*b*) External Source seen through an Emitting and Absorbing Cloud

A radio source of brightness B_S is observed through a cloud that emits as well as absorbs.



Change in brightness dB by a volume of length dr is

$$\frac{dB}{dr} = \frac{j\rho}{4\pi} - K\rho B$$

where, B is apparent (observed) brightness

ρ is density in kg/m^3, j is in watts/kg/Hz

K is absorption co-efficient in m^2/kg.

After solving, $B = B_S\, e^{-\tau_c} + \dfrac{j}{4\pi K}\left(1 - e^{-\tau_c}\right) = B_S\, e^{-\tau_c} + B_i\left(1 - e^{-\tau_c}\right)$

where, $\tau_c = \int_0^{r_1} K\,\rho\, dr$ is optical depth, or nepers attenuation by cloud of physical thickness r_1.

Using the Rayleigh-Jeans law, $T_b = T_S\, e^{-\tau_c} + T_c\left(1 - e^{-\tau_c}\right)$

where, T_S = source's temp., T_b = observed temp., T_c = cloud's temp.

Brightness and Antenna Temperature

Radio telescopes are instruments for measuring brightness of astronomical radio sources. A relation must be established between the power output from an antenna when pointed towards the source. The output from the antenna depends on:

(*i*) Brightness distribution of the source.

(*ii*) Shape of the radio source.

(*iii*) Brightness distribution of the sky.

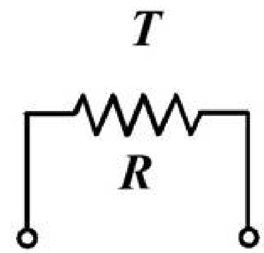
(*iv*) Beam shape of the antenna.

(*v*) Various efficiencies of the antenna.

(*vi*) Antenna pointing.

Brightness and Antenna Temperature

Consider a resistor R at temperature T. If $\Delta\nu$ is the bandwidth in Hz, then power W available at its terminals (in watts) is given as:



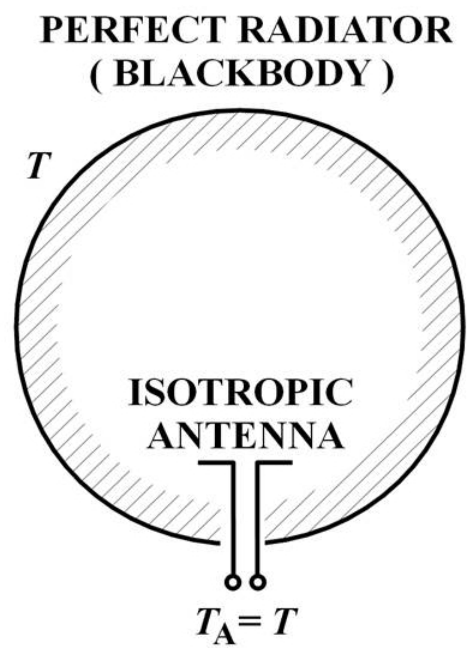
$$W = kT\Delta\nu$$ where, $k = 1.38\text{x}10^{-23}$ J/K (Boltzmann const.)

The spectral power w per unit bandwidth in watts/Hz is $w = kT$

Let a take a lossless matched isotropic antenna having a radiation resistance $R_r = R$. The power at it terminals will be:

(*i*) Zero if the antenna doesn't receive any radiation

(*ii*) Greater than zero if radiation is received.



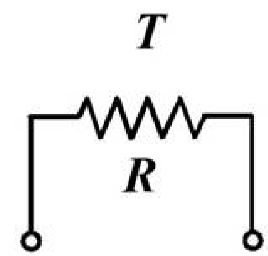
If we place this antenna inside a black body at a temperature T, so that it collects only the radiation and converts into spectral power w_A, we find

$$w_A = w \quad \text{or,}\ kT_A = kT \quad \text{i.e.,}\ T_A = T$$

where, T_A is known as the ***antenna temperature***.

Brightness and Antenna Temperature

Spectral power w across R is $w = kT \quad \ldots(1)$

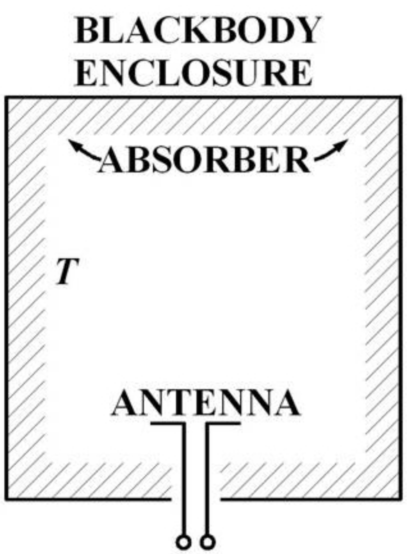


Consider a real antenna with effective aperture A_e and a normalized pattern $P_n(\theta,\varphi)$. The spectral power w is:

$$w = \frac{1}{2} A_e \iint B(\theta,\phi) P_n(\theta,\phi)\, d\Omega \quad \ldots(2)$$

Place the antenna inside a black body having a temperature T. By Rayleigh-Jeans law, a black body at temperature T has a constant brightness B_c for any (θ,φ).

i.e., $B(\theta,\phi) = B_c = \dfrac{2kT}{\lambda^2} \quad \ldots(3)$



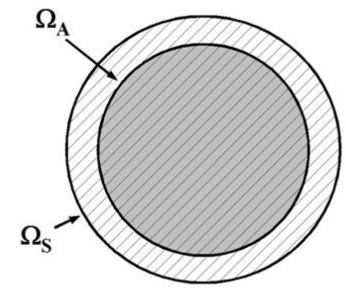
The antenna receives this brightness and produces power accordingly. This is obtained by putting equation (3) in (2).

i.e., $w = \dfrac{kT}{\lambda^2} A_e \Omega_A \quad \ldots(4)$

Since $A_e \Omega_A = \lambda^2$, we get $w = kT$, which is identical to (1).

Brightness and Antenna Temperature

The inner walls of the black body may be thought as a uniformly radiating celestial sphere with the antenna at its center. Thus the angular extent of the antenna lobes is less than the source extent. Recall that when source extent is greater than the major lobe, the flux density S_0 observed (using antenna) is given as:



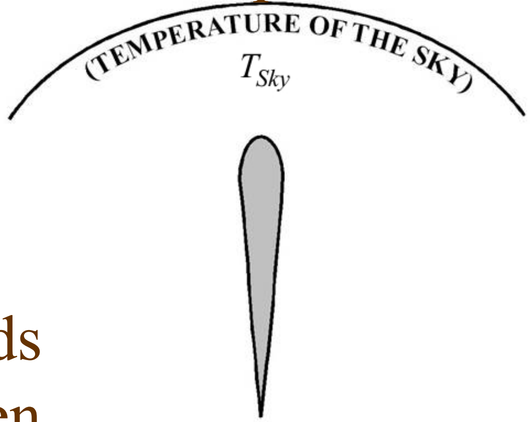
Source extent > major lobe

$$S_0 = B(\theta,\phi)\iint P_n(\theta,\phi)\, d\Omega \approx B(\theta,\phi)\,\Omega_M$$

Also we know that an antenna inside a black body produces spectral power

$$w = \frac{kT}{\lambda^2} A_e \Omega_A$$

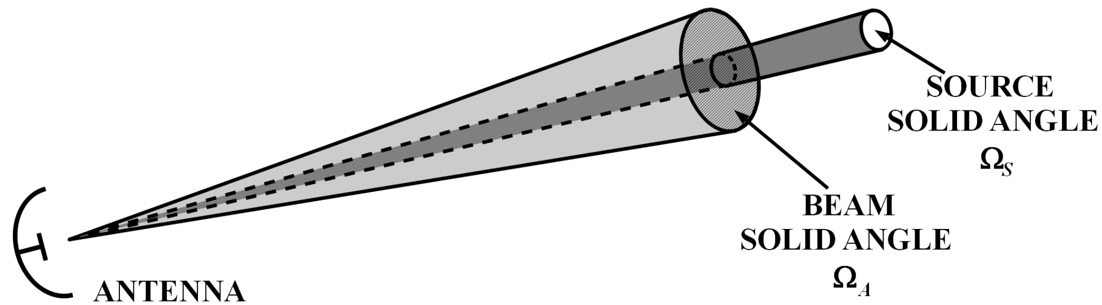
Relating the two we get: $S_0 = \dfrac{2kT_A}{A_e}$



<u>**Conclusion**</u>**:** If we point an antenna beam towards the sky, and the sky has a uniform brightness, then the antenna temperature $T_A = T_{Sky}$ (sky temp.).

Brightness and Antenna Temperature

Radio sources are small (discrete). If it is smaller than beam extent Ω_A, only a portion of the beam is occupied by the source solid angle Ω_S. The rest of the beam gets contribution of background sky possibly consisting of many other sources. Let the sky is at uniform temperature. Let the antenna temperature be $T_{Sky+Src}$.



Let us now point the antenna slightly away from the source (open sky). Let the corresponding antenna temperature measured is T_{Sky}.

Incremental temperature is $\Delta T_A = T_{Sky+Src} - T_{Sky}$

Source flux density will be $S = \dfrac{2k\,\Delta T_A}{A_e}$

Source temperature $T_S = \dfrac{\Omega_A}{\Omega_S}\Delta T_A$

If temp. distribution of sky is $T_s(\theta,\varphi)$, $T_A = \dfrac{1}{\Omega_A}\int_0^{\pi}\int_0^{2\pi} T_S(\theta,\phi)\,P_n(\theta,\phi)\,d\Omega$

Note: T_A is ***total antenna temperature***

Noise and Sensitivity of Radio Telescopes

The radio telescope system as a whole can be viewed as a combination of (*i*) antenna, (*ii*) transmission line and a (*iii*) receiver. Noise generated by these limits the sensitivity of the radio telescope. The causes of noise are:

(*i*) Antenna losses: mainly *antenna efficiency* and *impedance mismatch*

(*ii*) Cables losses: *attenuation* over length, *signal leakage* etc.

(*iii*) Connector losses: *attenuation*, small *mismatches* etc.

(*iv*) Receiver noise: Noise figure of fist stage of the receiver (LNA).

The combination of the above noise appears as a noise source at the terminals of the antenna called the ***system noise***. Its temperature equivalent is the ***system temperature***. The radio signal from the astronomical source should exceed this noise for detection.

In the following sections, we describe the calculations of the (*i*) system temperature and (*ii*) minimum detectable temperature.

System Temperature

T_A = Ant. noise temp (K)

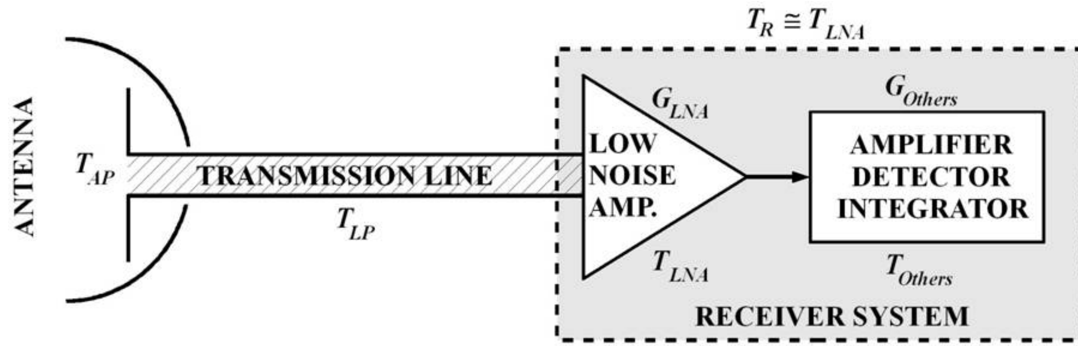
T_{AP} = Ant. physical temp (K)

ε_A = Thermal efficiency (< 1)

T_R = Rec. noise temp (K)

T_{LP} = Tx. line physical temp (K)

ε_{TxLi} = Tx. line efficiency (< 1). Dependent on length.



$\varepsilon_{TxLi} = e^{-\alpha l}$ where, α = Attenuation constant of tx. line (Np/m).

System temperature (K) is:

$$T_{Sys} = T_A + T_{AP}\left(\frac{1}{\varepsilon_A} - 1\right) + T_{LP}\left(\frac{1}{\varepsilon_{TxLi}} - 1\right) + \frac{1}{\varepsilon_{TxLi}} T_R$$

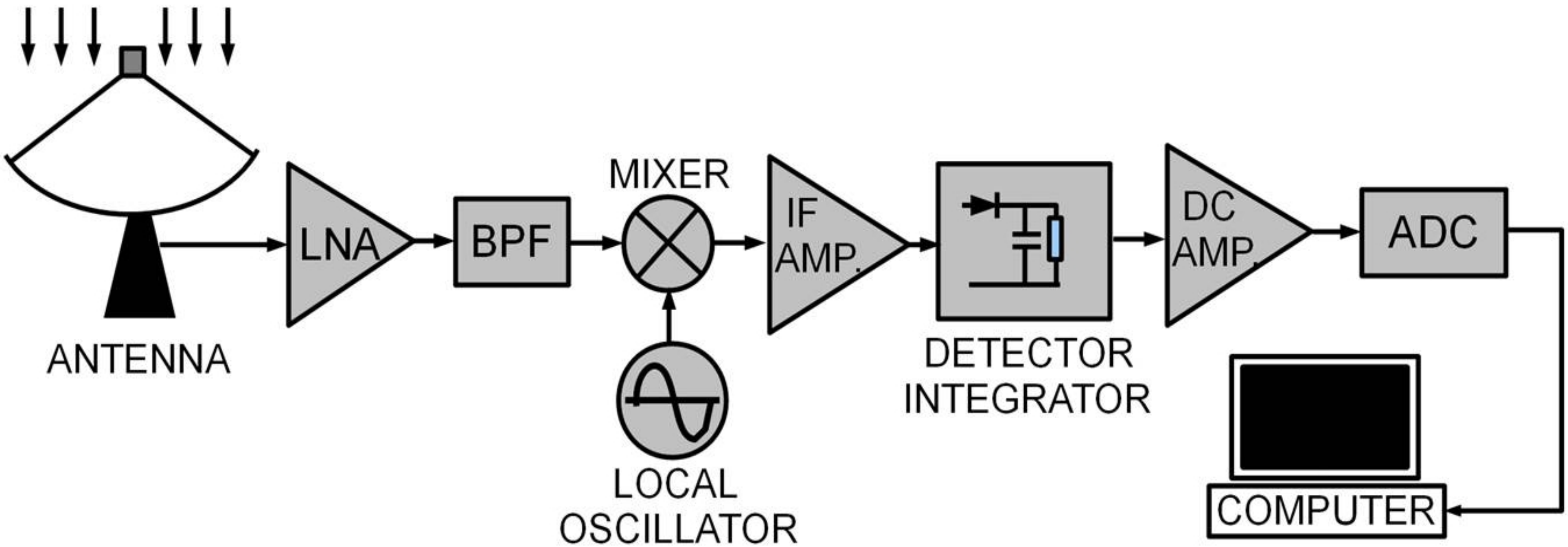
Receiver temperature (K) is:

$$\mathrm{T}_R = T_{LNA} + \frac{T_{Others}}{G_{LNA}} \simeq T_{LNA} \text{ if } G_{LNA} \gg \left(\frac{T_{Others}}{T_{LNA}}\right)$$

LNA temperature (K) is $T_{LNA} = (F-1)T_{LNA\_Phy}$

where, F is noise factor and T_{LNA_Phy} = Physical temp. of LNA (K).

Simple Heterodyne Radio Telescope



A basic heterodyne radio telescope receiver. The antenna, low noise RF amplifier (LNA), mixer, local oscillator, IF amplifier, detector, DC amplifier and data recording computer are shown.

Min. Detectable Temp., Brightness, Flux

Radio telescopes are sensitive only above a temperature ΔT_{min}, called ***minimum detectable temperature***. It depends on T_{Sys} which may be reduced by (*i*) increasing integration time, (*ii*) increasing pre-detection bandwidth $\Delta\nu$, or by (*iii*) taking the mean of two or more observations.

Caution: Large integration may distort true profile of the source. Large bandwidth results in (*i*) loss of spectral information and (*ii*) increase in local interference from terrestrial sources.

Min. detectable temp. (K) is: $$\Delta T_{\text{min}} = K_{Sys} \frac{T_{Sys}}{\sqrt{\Delta\nu\, n\, \tau}} = \Delta T_{\text{rms}}$$

where, K_{Sys} = system sensitivity constant (dimensionless).

Min. det. brightness (w/m^2/Hz/str) is:

$$\Delta B_{\text{min}} = \frac{2k}{\lambda^2}\Delta T_{\text{min}} = \frac{2k}{\lambda^2}\left(K_{Sys}\frac{T_{Sys}}{\sqrt{\Delta\nu\, n\, \tau}}\right)$$

where, n = no. of records averaged.

Min. det. flux density (w/m^2/Hz) is:

$$\Delta S_{\text{min}} = \frac{2k}{A_e}\Delta T_{\text{min}} = \frac{2k}{A_e}\left(K_{Sys}\frac{T_{Sys}}{\sqrt{\Delta\nu\, n\, \tau}}\right)$$

Brightness and Antenna Temperature

The output power from all practical antenna misses out some of the source's properties since the antenna beam pattern acts as a low pass filter on the source properties. We shall discuss these properties as:

(*i*) Convolution relationship between Antenna pattern and temperature distribution

(*ii*) Fourier Transform relationship between Antenna pattern and temperature distribution

(*iii*) Loss of Spectral Information and Aerial Smoothing

Brightness and Antenna Temperature

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(*iii*) Loss of Spectral Information and Aerial Smoothing

Ordinary frequency vs. Spatial frequency

Ordinary frequency $\nu = \frac{c}{\lambda}$

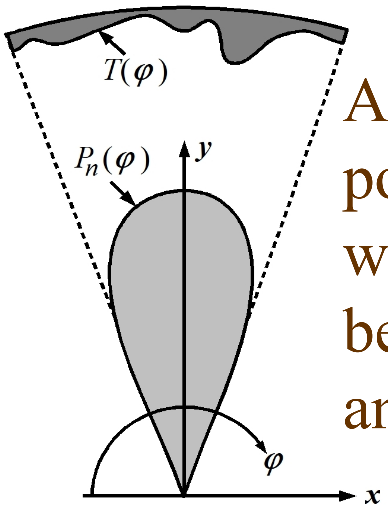
Spatial frequency $\nu_{spatial} = \frac{d}{\lambda}$

Here, c is speed of light (m/s), λ is wavelength and d is distance (m).

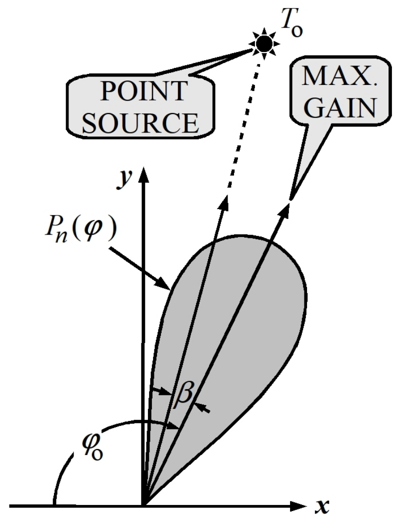
Antenna - Sky Temperature: Convolution

Convolution relationship in one dimension

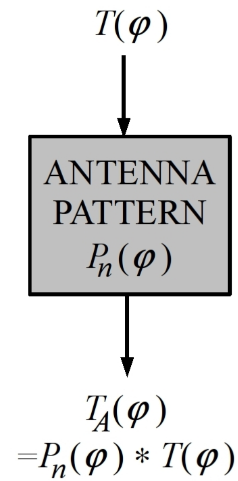
Power received by an antenna consists of (*i*) integrated power from the sky within antenna beam, modified by (*ii*) beam shape of antenna.



Absorbs power within beam angle.



Receives power from a point radio source at angle φ_0-β.



Relation of ant. temp., radiation pattern and src. temp. distribution.

$P_n(\varphi)$ = One dimensional normalized antenna pattern.

$T(\varphi)$ = One dimensional temperature distribution of sky.

T_0 = Temperature of the point source.

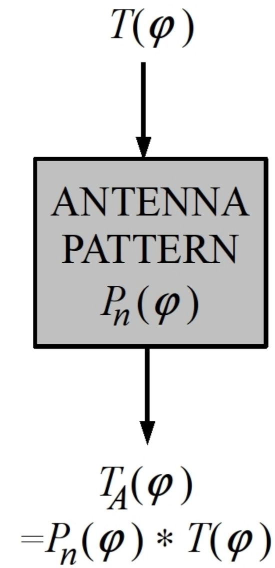
Antenna temperature of point source (at φ_0): $T_A(\varphi_0) = P_n(\varphi_0 - \beta)\, T_0$

Ant. temp. of extended source

$$T_A(\varphi) = \int P_n(\varphi - \beta)\, T(\beta)\, d\beta = P_n(\varphi) \star T(\varphi)$$

Antenna - Sky Temperature: Convolution

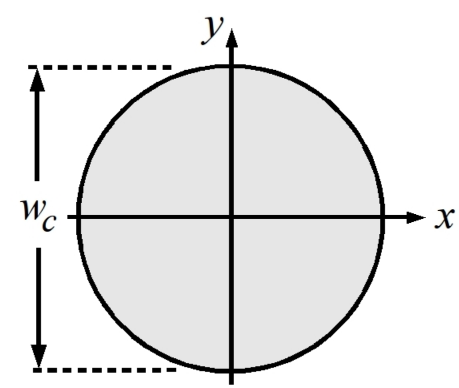
$$T_A(\varphi) = \int P_n(\varphi - \beta)\, T(\beta)\, d\beta = P_n(\varphi) \star T(\varphi)$$



Thus, the temperature output power from an antenna is the convolution of the normalized radiation pattern with the angular distribution of the source's temperature.

Convolution relationship in two dimensions

The antenna temperature, source temperature and the antenna pattern are two dimensional. Let the antenna aperture coincides with the *x*-*y* plane and centered at $(x,y) = (0,0)$. Using the dummy variables ξ and η we may represent the convolution as:



$$T_A(x,y) = \int_{-\infty}^{\infty} P_n(x-\xi,\ y-\eta)\, T(\xi,\eta)\, d\xi\, d\eta = P_n(x,y) \star T(x,y)$$

Ant. Patten - Temp: Fourier Transform

One dimensional Fourier transform relationship

We have seen in the spatial domain $T_A(\varphi) = P_n(\varphi) \star T(\varphi)$

Expressing it as a product in spectral domain (s): $\overline{T}_A(s) = \overline{P}_n(s)\ \overline{T}(s)$

where,

$$\overline{T}_A(s) = \int_{-\infty}^{\infty} T_A(\varphi)\, e^{-j2\pi s\varphi}\, d\varphi$$ i.e., FT of $T_A(\varphi)$

$$\overline{P}_n(s) = \int_{-\infty}^{\infty} P_n(\varphi)\, e^{-j2\pi s\varphi}\, d\varphi$$ i.e., FT of $P_n(\varphi)$

$$\overline{T}(s) = \int_{-\infty}^{\infty} T(\varphi)\, e^{-j2\pi s\varphi}\, d\varphi$$ i.e., FT of $T(\varphi)$

Highest freq. present in $P_n(\varphi)$, i.e. cut off frequency $s_c = \dfrac{w}{\lambda}$

where, w = ***aperture width*** along φ.

Cut-off period (inverse of s_c) $\varphi_c = \dfrac{1}{s_c} = \dfrac{\lambda}{w}$

Ant. Patten - Temp: Fourier Transform

Two dimensional Fourier transform relationship

We have seen that $T_A(x,y) = P_n(x,y) \star T(x,y)$

Expressing it in spectral domain (u,v): $\bar{T}_A(u,v) = \bar{P}_n(u,v)\,\bar{T}(u,v)$

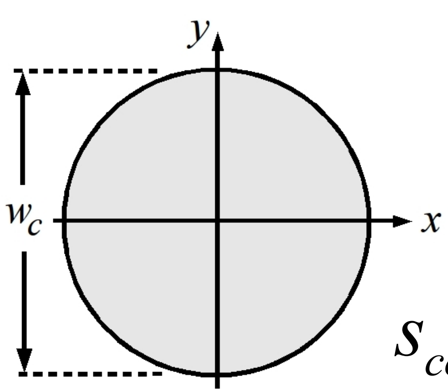
where,

$$\bar{T}_A(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T_A(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$ i.e., FT of $T_A(x,y)$

$$\bar{P}_n(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_n(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$ i.e., FT of $P_n(x,y)$

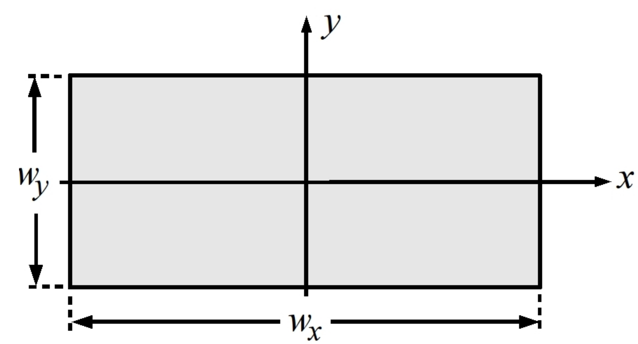
$$\bar{T}(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$ i.e., FT of $T(x,y)$

Cut off frequencies



$$s_{cc} = \frac{w_c}{\lambda}$$

Circular aperture



$$s_{cx} = \frac{w_x}{\lambda}$$

$$s_{cy} = \frac{w_y}{\lambda}$$

Rectangular aperture

Aerial Smoothing: Loss of Spectral info.

Consider a normalized antenna pattern: $P_n(\varphi) = k_1 \dfrac{\lambda}{w}\left[\dfrac{\sin(\pi\varphi w/\lambda)}{\pi\varphi}\right]^2$

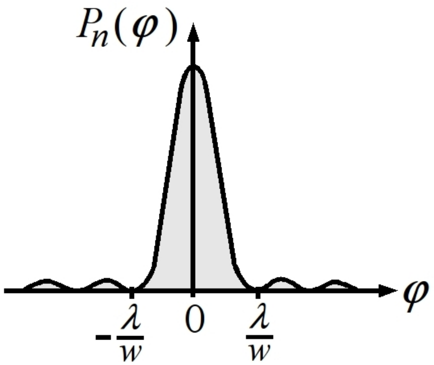
where, k_1 is const.

Also, $\int_0^{2\pi} P_n(\varphi)\,d\varphi = \varphi_A$

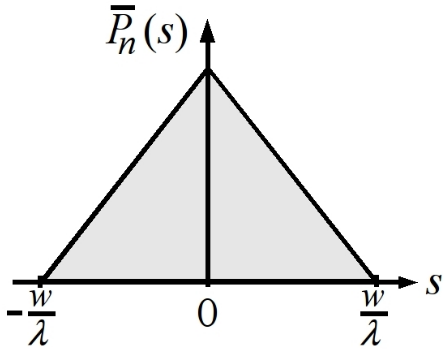
Beam-width between first nulls is $2\lambda/w$.

Half power beam-width is $0.89\ \lambda/w$.

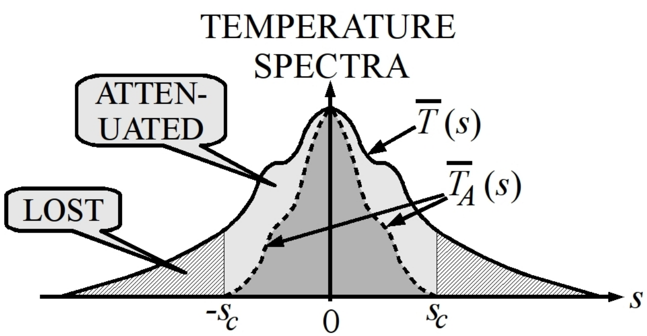
Below we plot theFourier transform relationship $\bar{T}_A(s) = \bar{P}_n(s)\,\bar{T}(s)$



Radiation pattern of an antenna.



Fourier transform of antenna pattern.

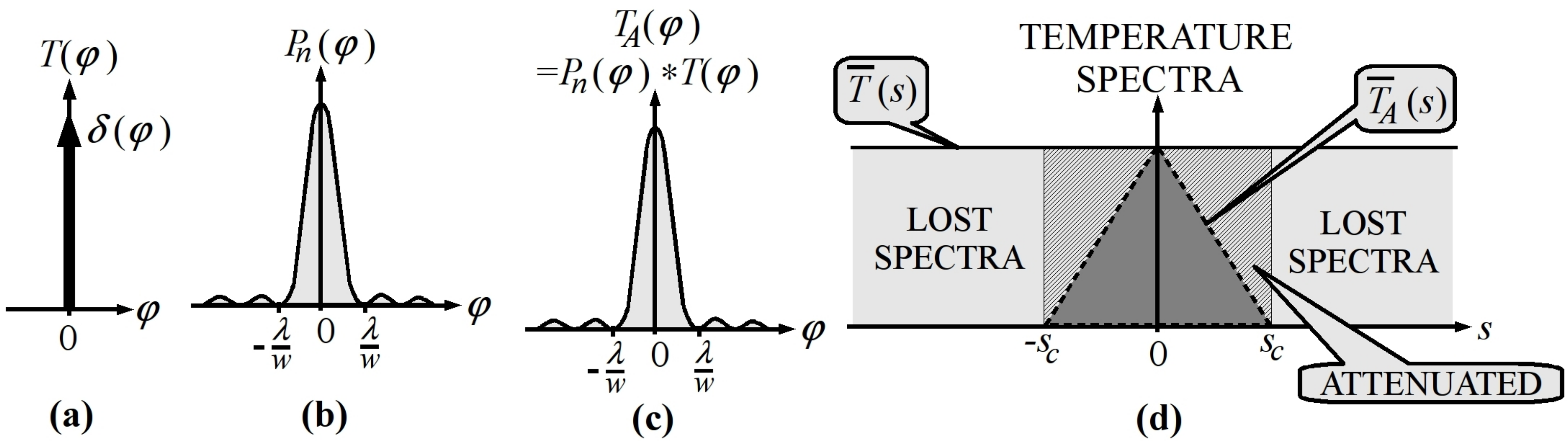


Low pass filtration of spectral components.

Since the Fourier transform of the radiation pattern ends at $s = \pm w/\lambda$ (i.e., $\pm s_c$), *spectral components beyond* $|s_c|$ *is not received by the antenna.* Also, *spectral components within* $|s_c|$ *are attenuated.*

Aerial Smoothing: Loss of Spectral info.

For a better understanding, consider a point source $\delta(\varphi)$ as shown:



Loss of spectral information from a point source.

(a) A point source represented by delta function of φ.

(b) Normalized radiation pattern of an antenna as a function of φ.

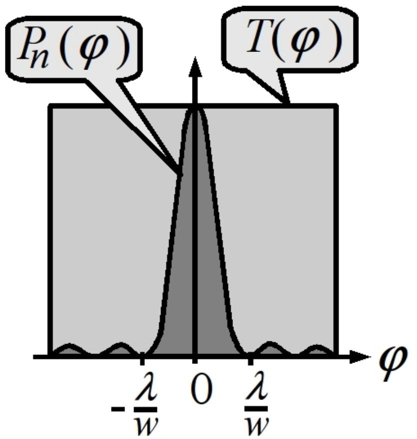
(c) Convolution of the delta function with the radiation pattern is identical to the radiation pattern itself.

(d) Temperature spectra of the point source and the antenna.

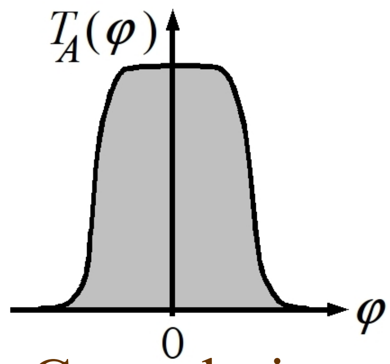
Remenber $\bar{T}_A(s) = \bar{P}_n(s)\,\bar{T}(s)$

Aerial Smoothing: Loss of Spectral info.

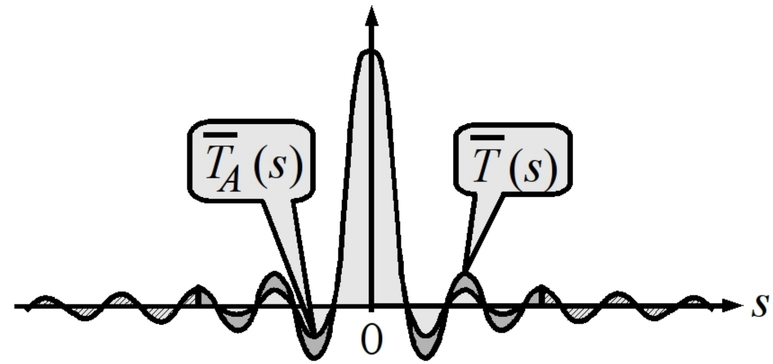
Rectangular Source



Patterns of source and antenna.

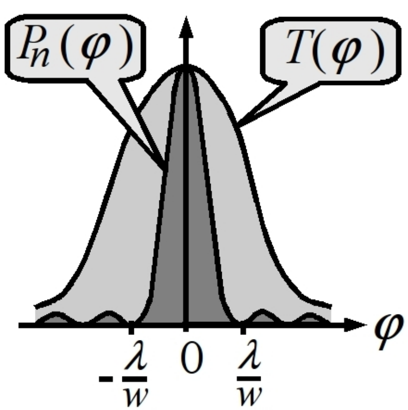


Convolution pattern.

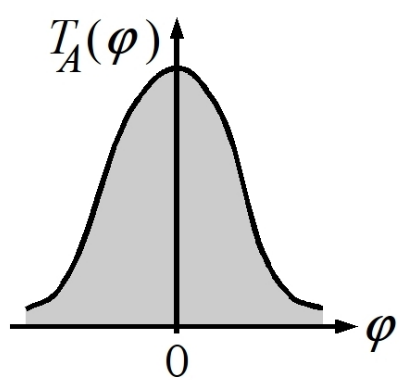


Source spectra and antenna temp. spectra compared.

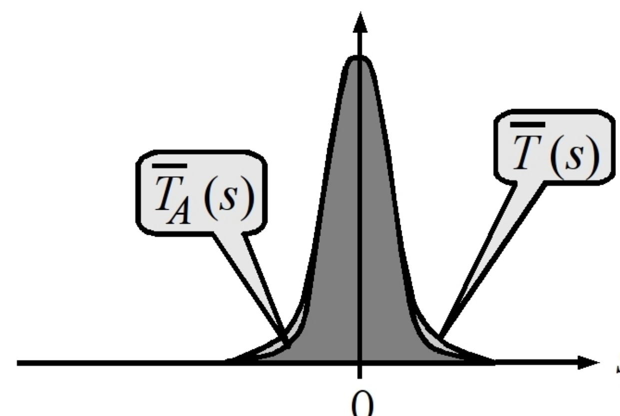
Gaussian Source



Patterns of source and antenna.



Convolution pattern.

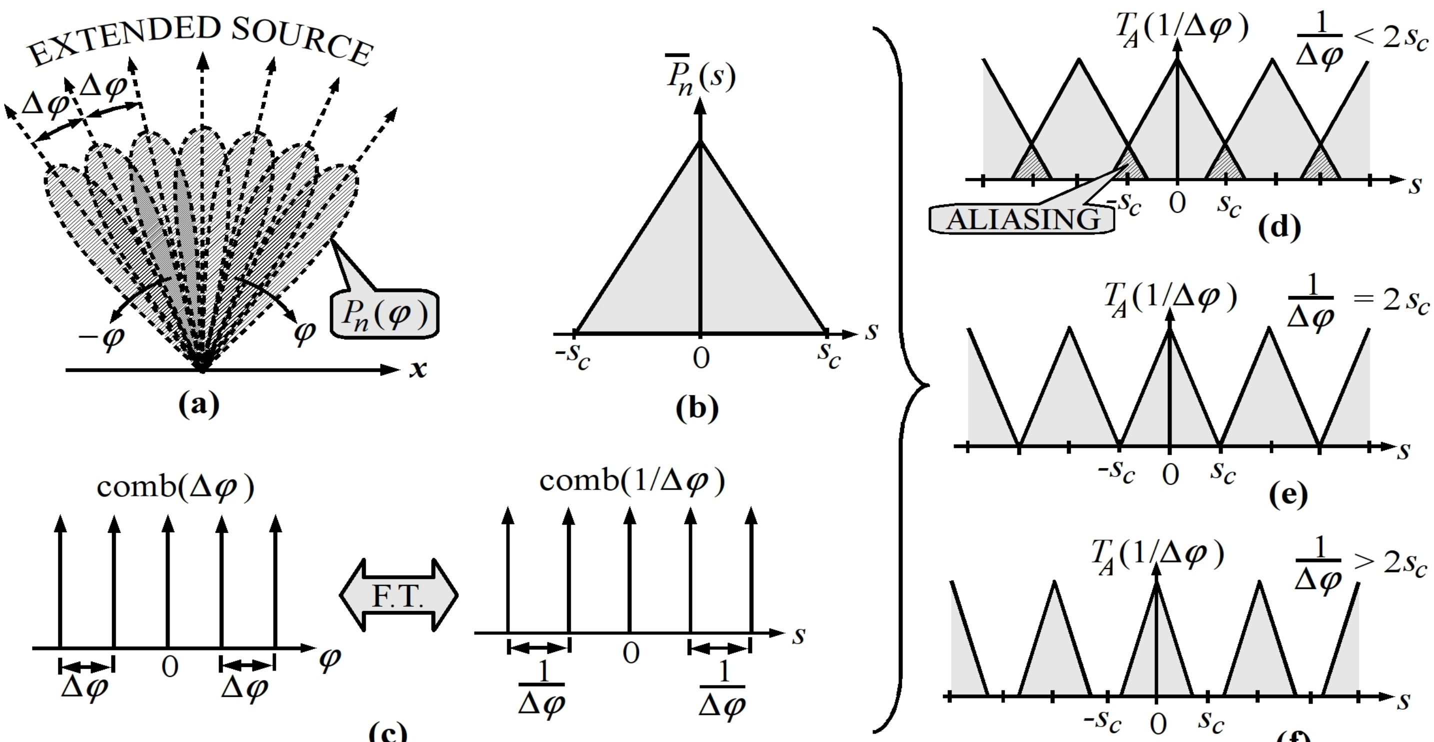


Source spectra and antenna temp. spectra compared.

Sampling Theorem of Observing Angle

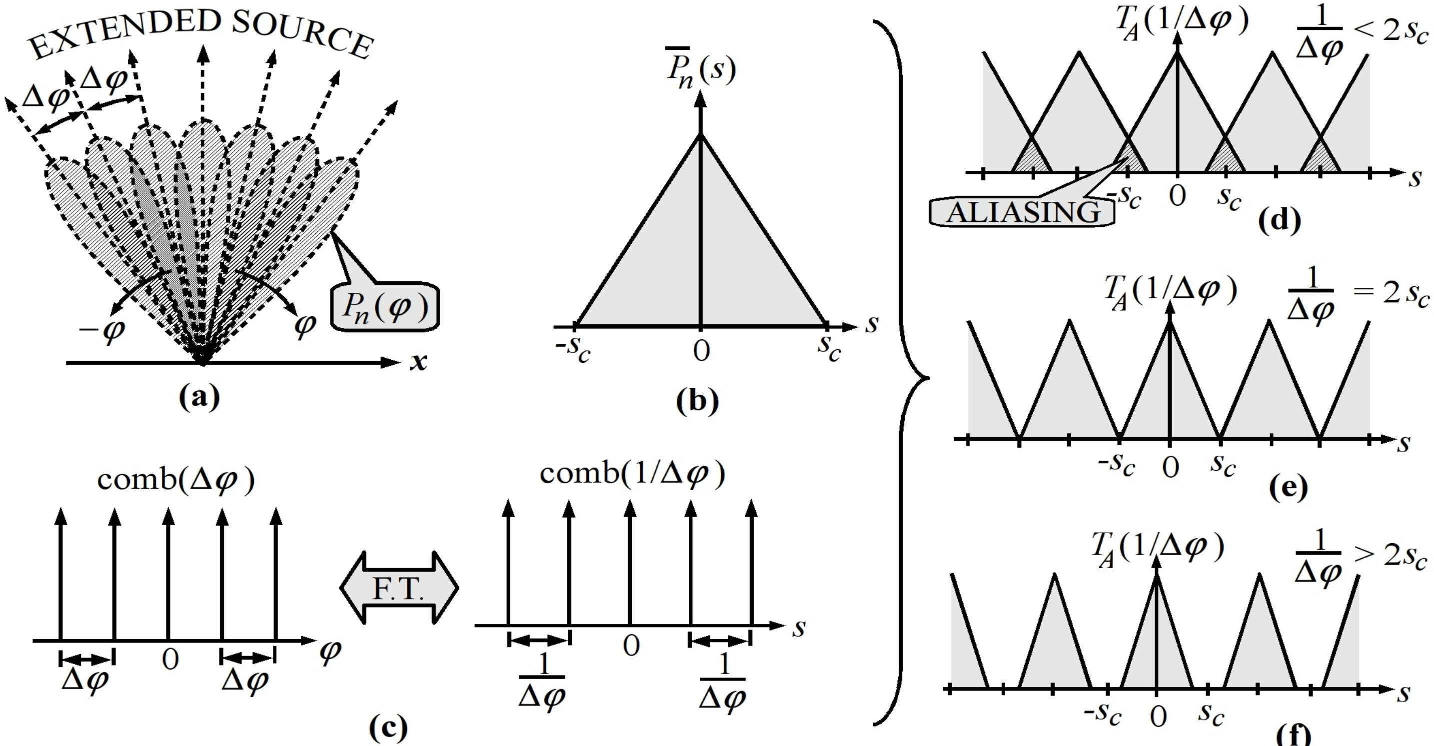
One dimensional Sampling Theorem

An observed distribution is completely determined by measurements spaced at equal discrete intervals which are at least as narrow as $1/2s_c$, where s_c is the *cut-off spatial frequency* of the antenna aperture.



(*a*) Scanning extended source at angular intervals $\Delta\varphi$. (*b*) Spatial antenna spectrum. (*c*) Discrete angular observation points and spatial Fourier transform. (*d*) Large angular scanning interval (aliasing). (*e*) Optimum scanning interval (true spectrum). (*f*) Small scanning interval (over sampling).

Sampling Theorem of Observing Angle



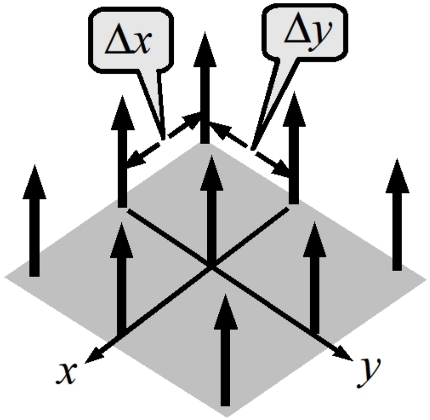
In angular domain, $\text{comb}\,(\Delta\varphi) = \sum_{m=-\infty}^{\infty} \delta\,(\varphi - m\,\Delta\varphi)$

Its Fourier transform, $\text{comb}\left(\frac{1}{\Delta\varphi}\right) = \sum_{m=-\infty}^{\infty} \delta\left(s - m\,\frac{1}{\Delta\varphi}\right)$

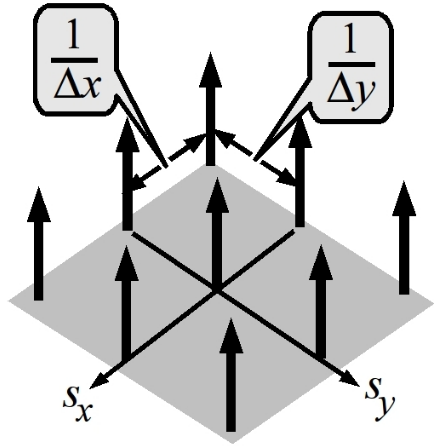
Ant. temperature spectrum $\overline{T}_A\,(s) = k_1\,\overline{P}_n(s) \sum_{m=-\infty}^{\infty} \delta\left(s - m\,\frac{1}{\Delta\varphi}\right) = k_1 \sum_{m=-\infty}^{\infty} \overline{P}_n\left(s - m\,\frac{1}{\Delta\varphi}\right)$

Sampling Theorem of Observing Angle

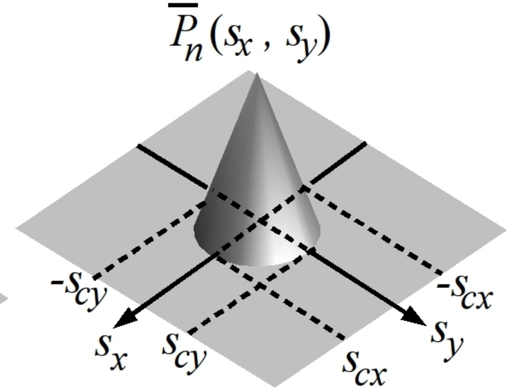
Two dimensional Sampling Theorem



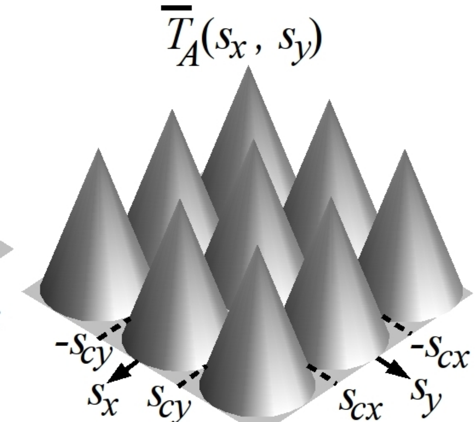
A two dimensional comb function.



Its spatial Fourier transform.



Two dimensional spatial spectral pattern of a single antenna.



Spectral pattern of synthesized antenna using multiple antennas.

In aperture domain,

$$\mathrm{comb}\,(\Delta x,\, \Delta y) = \sum_{m_x=-\infty}^{\infty} \sum_{m_y=-\infty}^{\infty} \delta_{\mathrm{II}}\,(x - m_x\,\Delta x,\;\; y - m_y\,\Delta y)$$

Its Fourier transform

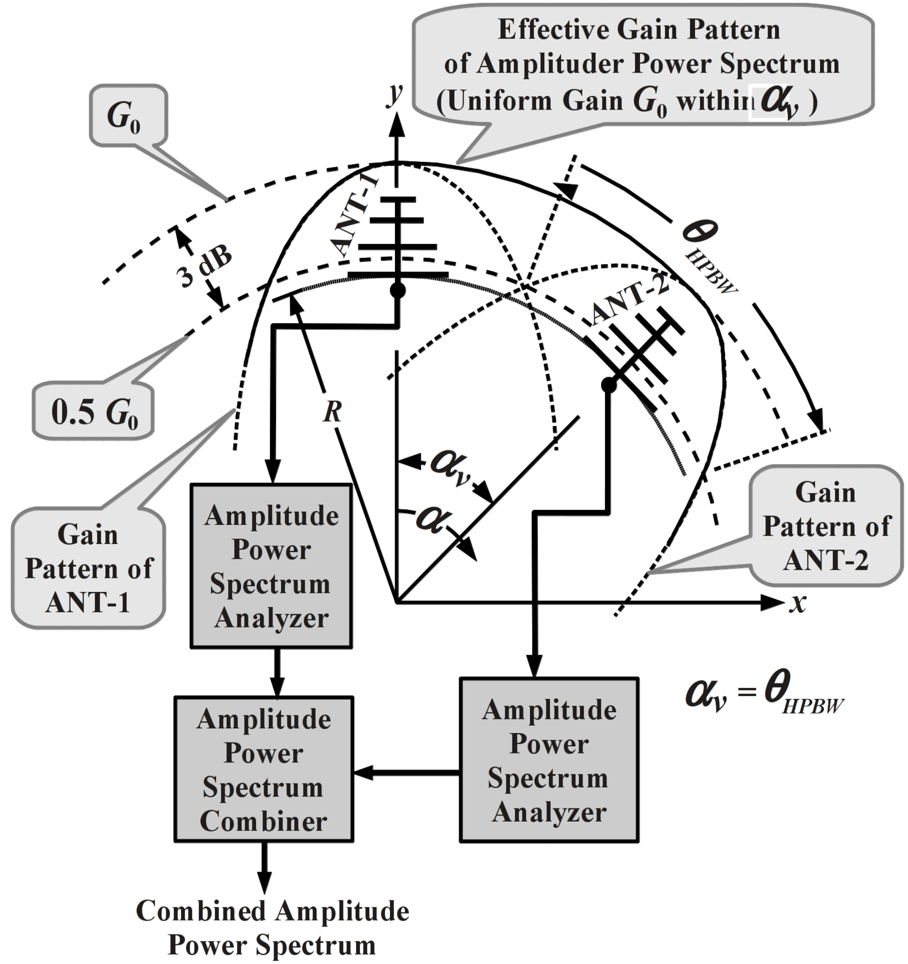
$$\mathrm{comb}\left(\frac{1}{\Delta x}, \frac{1}{\Delta y}\right) = \sum_{m_x=-\infty}^{\infty} \sum_{m_y=-\infty}^{\infty} \delta_{\mathrm{II}}\left(s_x - m_x\,\frac{1}{\Delta x},\;\; s_y - m_y\,\frac{1}{\Delta y}\right)$$

Ant. temperature spectrum

$$\overline{T}_A\,(s_x,\, s_y) = k_2 \sum_{m_x=-\infty}^{\infty} \sum_{m_y=-\infty}^{\infty} \overline{P}_n\left(s_x - m_x\,\frac{1}{\Delta x},\; s_y - m_y\,\frac{1}{\Delta y}\right)$$

Uniform Gain Pwr.-Spec. Ant.-Pattern Th.

If two electrically identical antennas possessing maximum individual gains G_0, *positioned in free space in a plane, such that they subtend an angle* α_v (equal to their HPBWs) *w.r.t. each other, then effectively the antenna system produces a magnitude power spectrum with uniform antenna gain* G_0 *and a uniform signal to noise ratio across the angle* α_v *if the magnitude power spectra of individual antennas are added.*

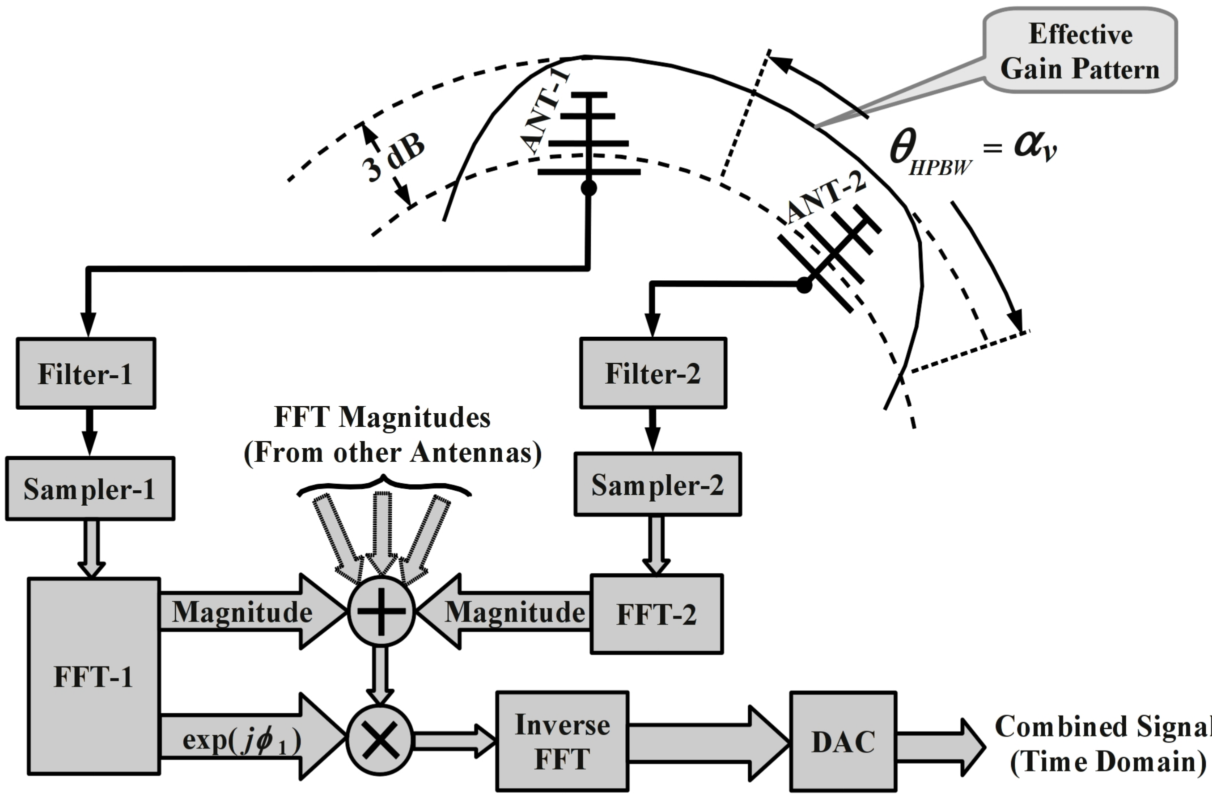


$$G(\alpha) = G_1(\alpha) + G_2(\alpha)$$

$$G(\alpha) = G_0[\cos^n(\kappa\alpha) + \cos^n\{\kappa(\alpha_v - \alpha)\}] \text{ where, } 0 \leq \alpha \leq \alpha_v,\ n \geq 1,\ \kappa \leq 1$$

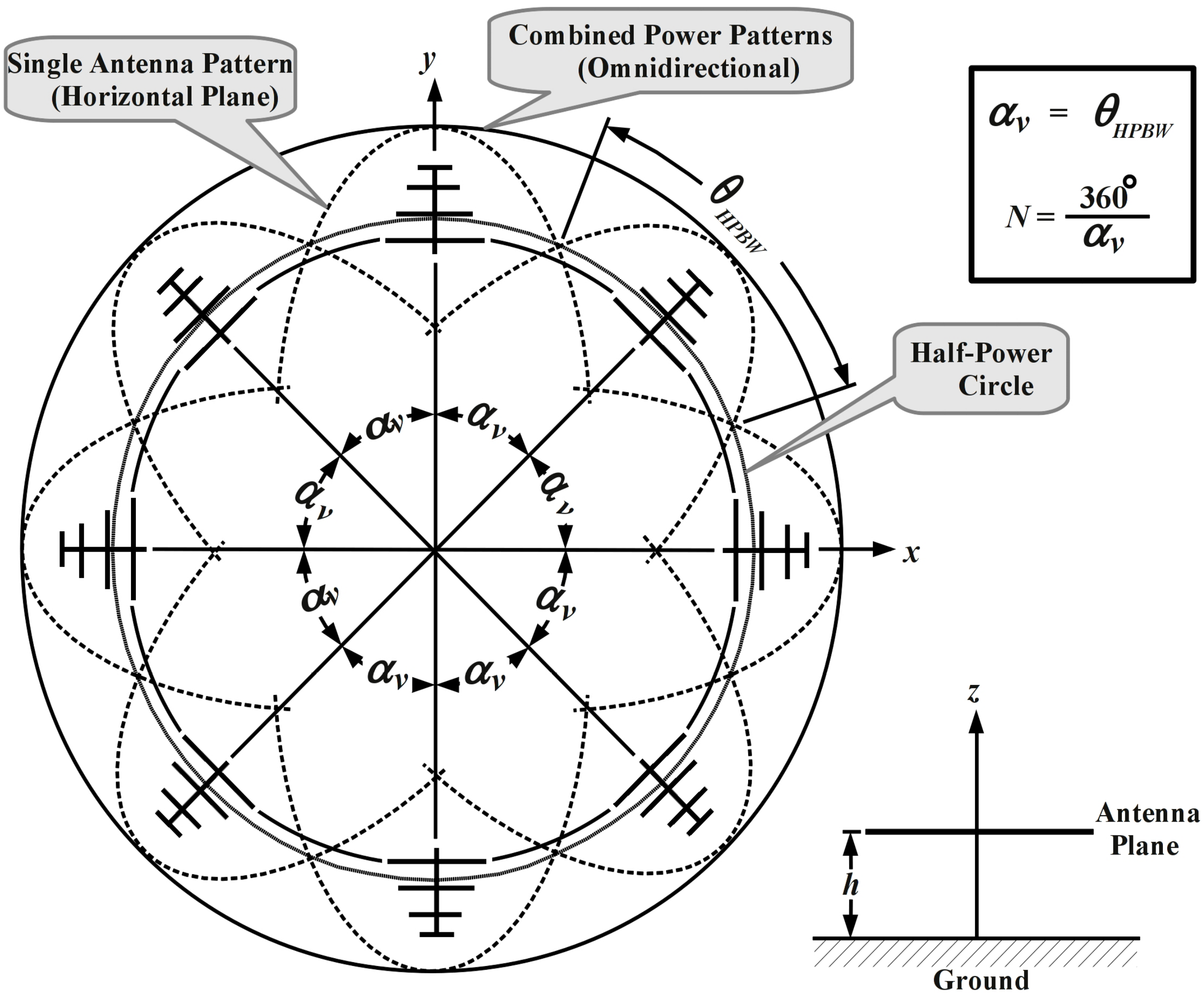
Useful for designing Spectrographs and RFI monitoring systems.

Uniform Gain Pwr.-Spec. Ant.-Pattern Th.



If the exponents of the imaginary phase values of every frequency channel obtained from one of the antennas are multiplied with the corresponding channel's sum of the amplitude spectrum and an inverse Fourier transform is applied, the time domain signal thus produced is effectively the signal that shall be obtained from a single antenna possessing a uniform gain G_0 across an angle α_v.

Uniform Gain Pwr.-Spec. Ant.-Pattern Th.



Several antennas of identical characteristics are used to form a complete omnidirectional power spectrum antenna pattern for RFI monitoring.

Assignment Problems-I

1. Explain the terms (*i*) *brightness*, (*ii*) *total brightness*, (*iii*) *total radio brightness* and (*iv*) *spectral power*.

2. Explain the term *radiance*. What is the relationship between radiance and brightness.

3. Define a *point source*, *localized source* and an *extended source*.

4. Write the equations for Planck's law of blackbody radiation in terms of (*i*) *frequency* and (*ii*) *wavelength*. Draw tentative curves of brightness verses frequency at a temperature of 5800 K.

5. Write the equations for Wien's displacement law. What is the wavelength at which the Sun has maximum intensity. Assume the temperature of the Sun as 5800 K.

Assignment Problems-II

6. Give the expressions of (*i*) Rayleigh Jeans law (*ii*) Wien's radiation law. Why are these laws called approximate laws?

7. Why is the Rayleigh Jeans law used for radio astronomy?

8. Explain the Stefan Boltzmann law with an expression. What is the total brightness of the Sun? Assume the temperature of the Sun to be 5800 K.

9. Compare the Rayleigh Jeans law and Wien's radiation law with the Planck's law of spectral radiance using a diagram.

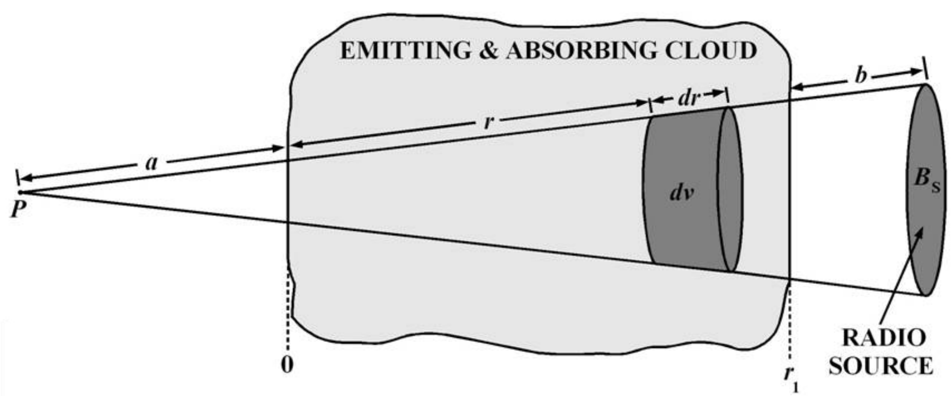
10. Explain with equations the meaning of (*i*) *optical depth* and (*ii*) *depth of penetration*.

Assignment Problems-III

11. Consider a source of brightness B_s observed through a cloud which emits as well as absorbs. The change in brightness dB produced over a volume length dr is given in Eq. Ex-11.1, where B is the apparent brightness, j is the emission coefficient, K is the absorption coefficient and $\tilde{n}$ is the density of matter within the cloud. Assuming a local thermodynamic equilibrium apply Kirchoff's law and show that the apparent brightness B can be expressed as in Eq. Ex-11.2.

Hint: $dB = 0$

$$dB = \text{emission} - \text{absorption} = \left(\frac{j}{4\pi} - B\,K\right)\rho\,dr \quad \text{...(Ex-11.1)}$$



$$B = \frac{j}{4\pi K} \quad \text{...(Ex-11.2)}$$

Assignment Problems-IV

12. A spherical object in the sky subtends an angle 0.049° across and emits like a blackbody. An antenna with a half power beam width of 0.115° is used to measure the temperature of the object. The measured incremental temperature is 0.239 K. Assume k_B to be 0.8 calculate the temperature of the spherical object.

Hint:
$$T_S = \frac{\Omega_A}{\Omega_S} \Delta T_A$$

$$\left.\begin{aligned} \Omega_M &= \iint_{\substack{\text{main}\\\text{lobe}}} P_n(\theta,\phi)\, d\Omega \approx k_B \theta_{HP}\, \phi_{HP} \\ &\text{where, } 0.8 \le k_B \le 1.0 \end{aligned}\right\}$$

13. Assuming the antenna efficiency as 100% and a wavelength of 3 cm, for above problem, calculate the source flux density.

Hint:
$$D = \frac{40000}{\theta_{HP}\, \phi_{HP}} \qquad D = 4\pi \frac{A_e}{\lambda^2} \qquad S = \frac{2k\,\Delta T_A}{A_e}$$

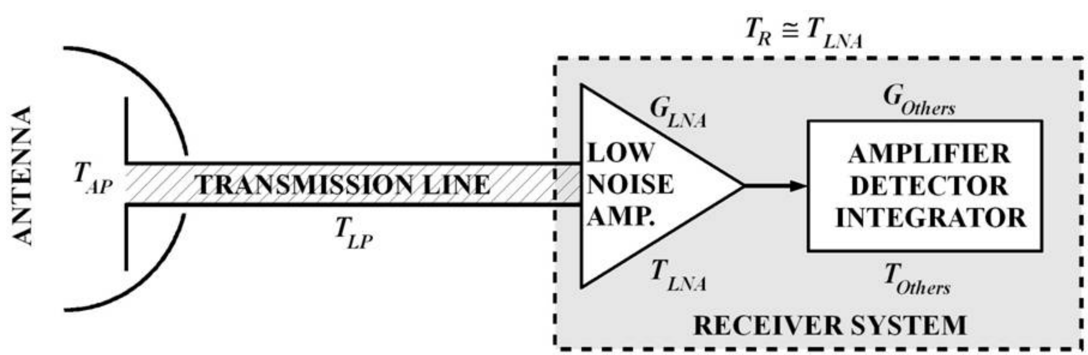
Assignment Problems-V

14. A single antenna radio telescope with the model shown is operated in room temperature (300 K). The length of the transmission line between the antenna and the receiver is 2 m and has an attenuation constant of 0.05 Nepers per meter. The noise temperature from the antenna is 55 K and the efficiency of the antenna is 98%. The receiver temperature is 70 K. Find the system temperature.

Hint:

$$T_{Sys} = T_A + T_{AP}\left(\frac{1}{\varepsilon_A} - 1\right) + T_{LP}\left(\frac{1}{\varepsilon_{TxLi}} - 1\right) + \frac{1}{\varepsilon_{TxLi}} T_R$$

$$\varepsilon_{TxLi} = e^{-\alpha l}$$



Assignment Problems-VI

15. A low noise amplifier (LNA) operates at room temperature (300 K) having a noise factor of 1.2. Calculate noise temperature of the LNA.

Hint: $$T_{LNA} = (F-1)T_{LNA\_Phy}$$

16. The gain of the LNA described in problem above is 10000, and the temperature contribution from rest of the system following the LNA is 500 K. Find the receiver temperature and compare with the temperature of the LNA.

Hint:
$$T_R = T_{LNA} + \frac{T_{Others}}{G_{LNA}}$$
$$\simeq T_{LNA} \text{ if } G_{LNA} \gg \left(\frac{T_{Others}}{T_{LNA}}\right)$$

Assignment Problems-VII

17. A radio telescope has a system temperature of 90 K. The system sensitivity constant is 0.7 and the integration time is 1 second. The number of averaged records is 20 and the operating frequency of the radio telescope is 1420 MHz using a bandwidth of 30 MHz. Calculate the (*i*) minimum detectable temperature, (*ii*) minimum detectable brightness and the (*iii*) minimum detectable flux density.

Hint:

$$\Delta T_{\min} = K_{Sys} \frac{T_{Sys}}{\sqrt{\Delta\nu \, n \, \tau}} = \Delta T_{\mathrm{rms}}$$

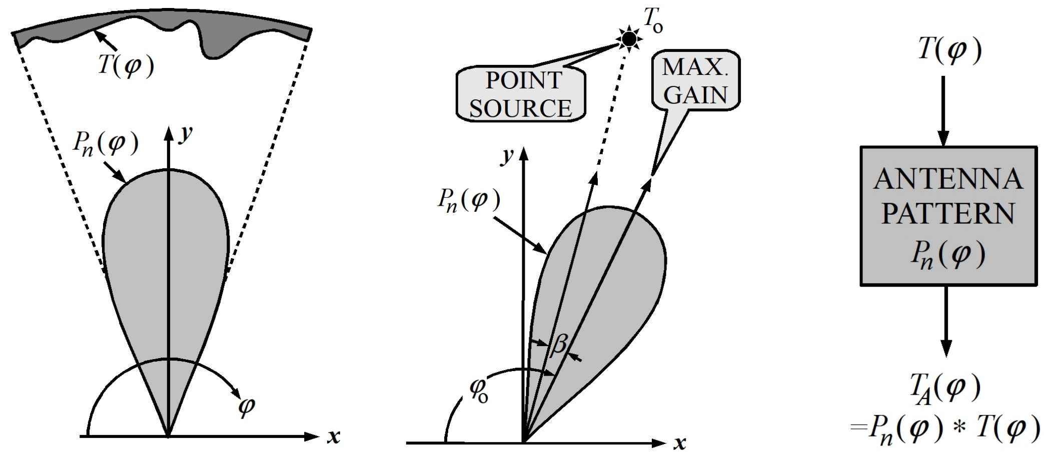
$$\Delta B_{\min} = \frac{2k}{\lambda^2} \Delta T_{\min} = \frac{2k}{\lambda^2} \left(K_{Sys} \frac{T_{Sys}}{\sqrt{\Delta\nu \, n \, \tau}} \right)$$

$$\Delta S_{\min} = \frac{2k}{A_e} \Delta T_{\min} = \frac{2k}{A_e} \left(K_{Sys} \frac{T_{Sys}}{\sqrt{\Delta\nu \, n \, \tau}} \right)$$

Assignment Problems-VIII

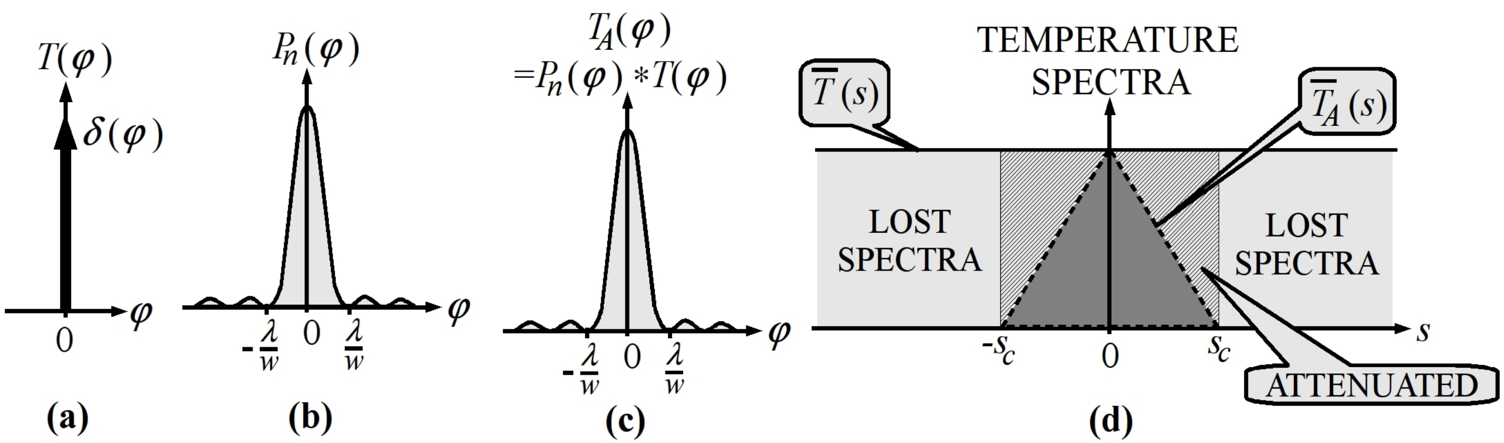
18. How are the antenna temperature, antenna pattern and related?

Hint: See Fig.



19. What is aerial smoothing? Explain graphically using simple equations how the loss of temperature spectra occurs due to low pass characteristics of the antenna pattern.

Hint: See Fig.

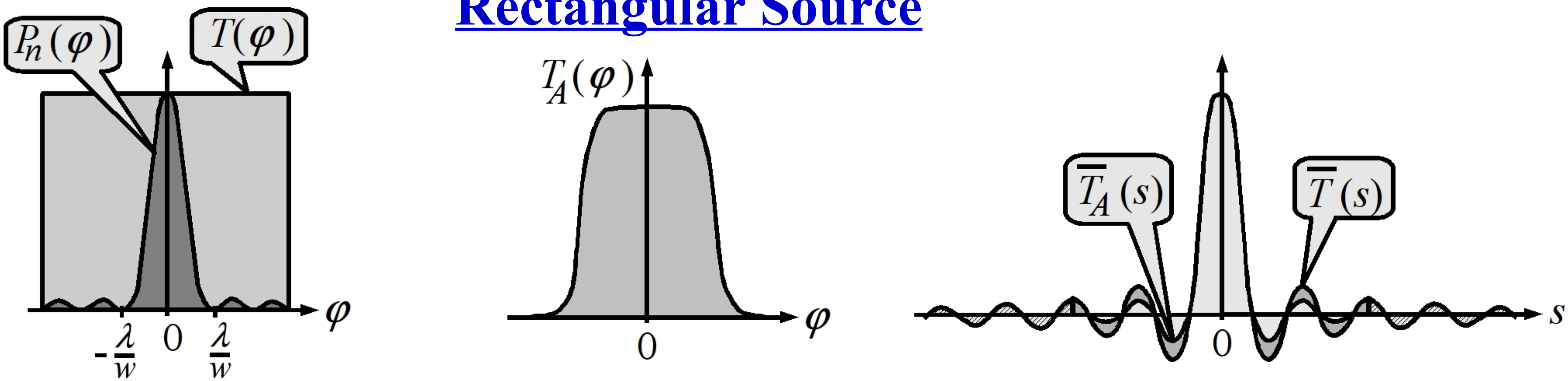


Assignment Problems-IX

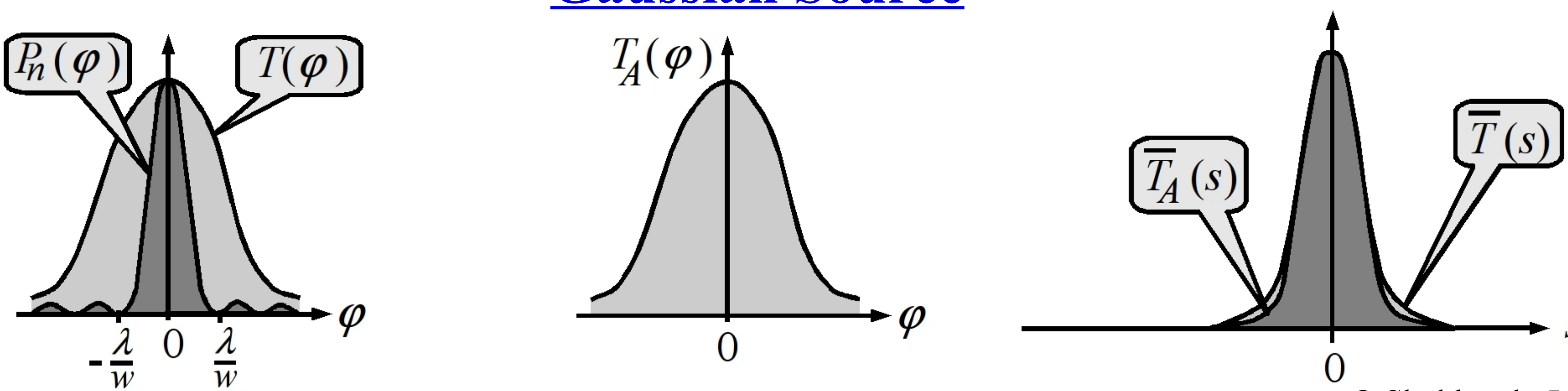
20. Justify with simple reasons why loss of spectral components are less when observing an extended Gaussian source than an extended rectangular source.

Hint: See Fig.

Rectangular Source



Gaussian Source



Assignment Problems-X

21. State and expain the one dimensional sampling theorem of observing angle. Where is its application?

22. Explain the two dimensional sampling theorem of synthesized aperture. Where is its application?

23. Explain the Uniform Gain Power-Spectrum Antenna Pattern Theorem.

THANK YOU